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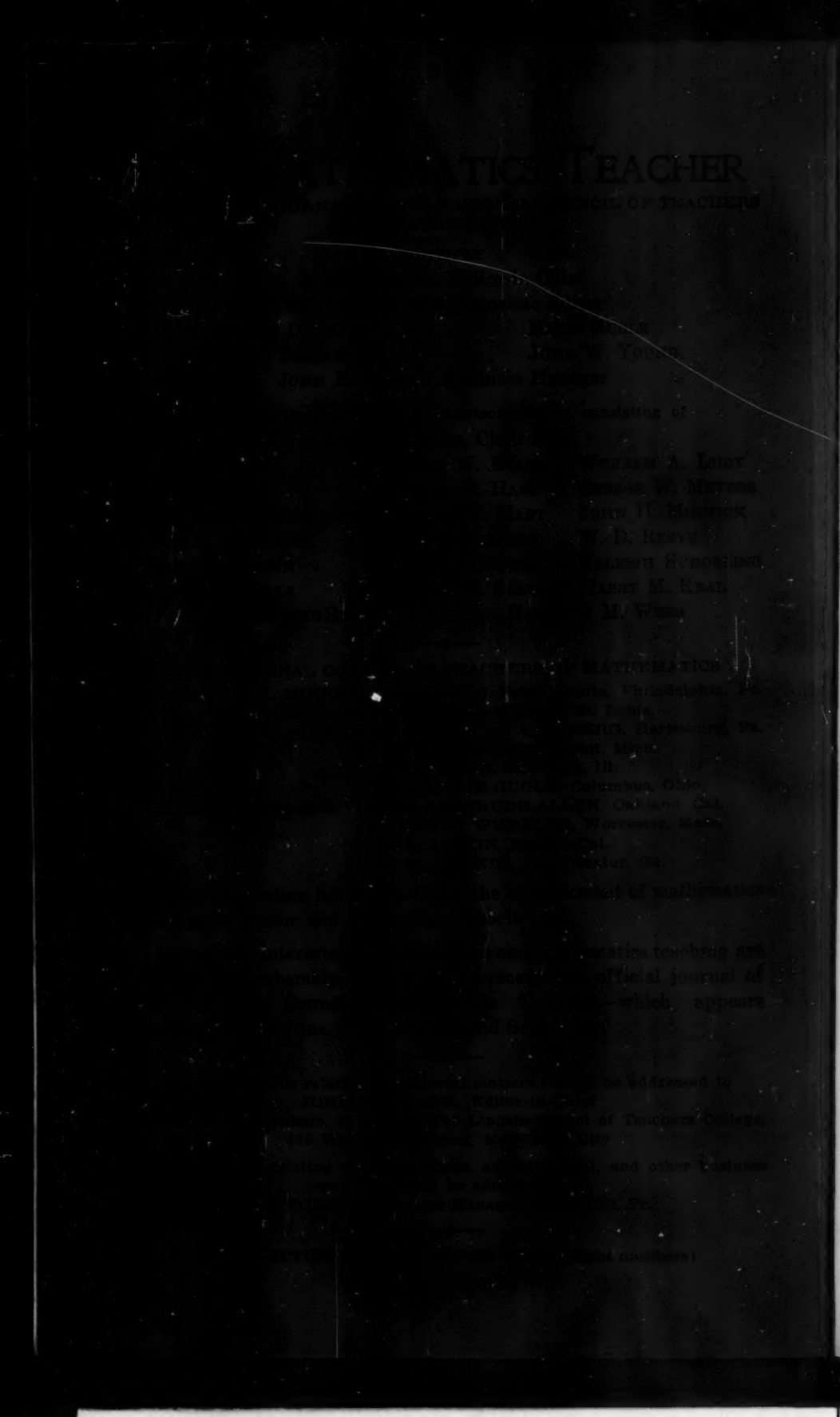

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THE MATHEMATICS TEACHER

VOLUME XVII JANUARY, 1924 NUMBER 1

CURRICULUM MAKING: WHAT SHALL CONSTITUTE THE PROCEDURE OF NATIONAL COMMITTEES?

HAROLD RUGG
The Lincoln School of Teachers College

American education can advance no more rapidly than the rate at which its basic facts and laws are discovered. Within the field of the mathematics curriculum two concurrent publications exhibit clearly the chief methods by which school men are attempting to discover these basic facts and laws. These publications are: (1) Thorndike and others, *The Psychology of Algebra*[1]; (2) *The Reorganization of Mathematics in Secondary Education*[2] (Mathematical Association of America), prepared by a committee of mathematics teachers.

Students of curriculum-making, as well as teachers of mathematics, should study these two reports together. They supply notable additions to our knowledge of the content and teaching of secondary mathematics. But better than that, for the purposes of our discussion, they set out in clear relief two sharply contrasted methods of improving the materials of instruction in American schools—the method of committee procedure and the method of laboratory analysis. These two methods are in reality complementary. Neither should be ignored. One, the fundamental one, is being ignored to such an extent that the situation should be commented upon somewhat fully.

The Reorganization of Mathematics in Secondary Education is the report of thirteen teachers of mathematics, working as the National Committee on Mathematical Requirements. *The Psychology of Algebra*, on the other hand, is the report of the investigations of Dr. Thorndike and five co-workers in the Institute of Educational Research of Teachers College. Thus we have at hand for study, excellent examples of two methods of work: the method of expert teaching opinion and the method of expert analysis of social and psychological needs. One is

[1] Macmillan Company, New York City.
[2] Distributed by Professor J. W. Young, Hanover, N. H.

primarily subjective, the other much more objective. Particular phases of the total problem can be most effectively attacked by the former; without the latter, however, no lasting progress in curriculum-making can be made.

It is fortunate for our discussion that these reports are good examples of the two methods under consideration. There is no question that they are. Years of critical study of committee procedure leads to the conclusion that the report of the National Committee is far superior to any report hitherto produced in any school "subject" by such methods. Its methods, too, were much more systematic and thorough than those of earlier committees.

Yet in spite of the thoroughness and intelligence with which the work of the committee was carried out, its report shows the inadequacy of methods of committee procedure which are not based primarily on objective analysis. It is chiefly to a review of the methods employed that we shall address our comments.

Two preliminary comments should be made to orient the reader of this article. First: the purpose of this paper is not to present a detailed review of all that the two groups did. That is impossible in the space at hand. The reports are used here merely because they supply clear illustrations of the techniques that should be employed by National Committees. Second: it is especially important to keep that point in mind because of the difference in the scope of the curriculum covered by the two reports. The studies of Professor Thorndike and his group are centered upon algebra in the ninth grade; those of the National Committee deal with the entire range of secondary mathmatics. There is no intent to make comparisons which are based upon this difference in the scope of the curriculum with which each group dealt.

TWO TYPES OF EXPERT SERVICE ARE NEEDED BY NATIONAL COMMITTEES

There is one significant characteristic about this latest enterprise in curriculum-making in mathematics: the committee was adequately financed. This is probably the most important fact before us today: that for the first time in the history of education funds are available for the study and improvement of school processes. It appears therefore, that the National Committee

had an unusual opportunity. It could employ expert service.

Two types of expert service were needed. In the order in which they could be utilized they were (1) that of specialists in curriculum-making, that is specialists trained in the study of intelligence and pupils' interests, in the study of social needs and in the analysis of learning; (2) that of teachers of mathematics in colleges and secondary schools who were trained in the authentication of mathematical subject-matter, and were sensitive to the temper of teachers and the needs of the class-rooms. The first group of specialists is needed to discover the fundamental facts and laws of intelligence, social needs and learning. The second group is needed to organize the facts and laws into a working program of class-room instruction. The tasks required of each group demand specialized interests, training and experience. Neither group, as educational workers are now constituted, is fitted to do the work of the other. Both are necessary for the proper reorganization of mathematics; *both are indispensable in the proper reorganization of any department of school work.*

THE NATIONAL COMMITTEE EMPLOYED ONLY ONE KIND OF EXPERT SERVICE

The National Committee, having large funds at its command, employed the second type of service. It did not employ specialists in curriculum-making. Part of its money was wisely used to secure the full-time services of two outstanding leaders in college and secondary mathematics—Professor J. W. Young of Dartmouth and Mr. J. A. Foberg of the Pennsylvania State Department of Public Instruction. These two men maintained efficient offices for two years and set up machinery by which they were able to coordinate the efforts of fully one hundred sectional organizations of teachers. They travelled much and widely. They "felt the pulse" of mathematics classrooms all over the country. They studied the literature of "the reorganization of mathematics" carefully. For the first time in the history of committees the analysis of school practice was oriented by a grasp of the current dynamic and inductive point of view in educational psychology. Their report reflects the distinctive success with which they were able to make themselves familiar with current thought in psychology. They gave unsparingly of

their energy and their intelligence in the attempt to sound the temper of teahcers and to evaluate the proposals of innovators. They organized the results of this inventory of what the country was willing to do in a program of grade and classroom instruction that advances very much the organization and teaching of mathematics. So well did they do what they regarded as their sole function that this is nothing short of a *conspicuous* committee report. *Certainly in no subject has so complete, so forward-looking a report come from the work of a national committee.* It is a clear-cut statement of the extent to which teachers of mathematics can be expected to put into practice a reorganized mathematical instruction.

We must not forget, however, that whereas two types of expert service were needed only one was employed—the traditional one—that of teachers trained and exprienced in subject-matter. This is typical of current practices in all curriculum-making in American schools. To the present time school courses of study have been made either by specialists in the authentication and documentation of subject-matter or by secondary school teachers who were trained by such people and are dominated by their points of view. These latter ones have frequently complained of late that "educationists" have interjected themselves into the work of making the course of study. Why, one hears it asked, should a person be permitted to reorganize the elementary and secondary mathematics curriculum when he is not primarily a mathematician? How can a person who is not a "geographer" construct a curriculum in geography, or how can people who have never published the results of historical research contribute to the school curriculum in history? "Why don't these pedagogues attend to their own jobs and leave our field alone," is the burden of the plaint.

The direct and significant reply is: that it is the specialists in subject-matter (at least those who have not entered definitely upon the study of experimental education) who are out of their proper territory when they attempt to do the fundamental work underlying the making of curriculum, not the students of the science of education. Curriculum-making is rapidly becoming a specialized technical field of work, success in which requires scientific knowledge of society and how to study it, a grasp of the facts and laws of human behavior and definite training and

experience in the use of scientific tools of education—namely, objective analysis, measurement, statistical methods and the technique of experimentation. The fact is that we are witnessing the development of a new body of specialists who are devoting their energies to the building up of a scientific approach to the curriculum. Already the work of this group is influencing fundamentally certain phases of elementary school instruction. (For example, what book company will now publish a spelling book without checking its content against the results of the most authoritative scientific studies of adult writing vocabularies?)

Is it not clear that the task of curriculum-making is a specialized technical task to be carried on most effectively by a body of curriculum specialists? Should not these be trained, on the one hand, in the study of social needs, in the analysis of the abilities and interests of growing children and in the experimental study of learning; and on the other hand in testing the temper of school teachers and administrators; that is of finding out what the country will do? The second task amounts to getting school officials to put into operation a curriculum based as nearly as possible on the finding of objective investigation. Since this would mean in most instances a very radical departure from the current order and since the curriculum-fitters are themselves a vital part of the current order the difficulties of the task will clear at the outset.

THE MAJOR QUESTIONS WHICH CONFRONT THOSE WHO PROPOSE REORGANIZATION OF THE CURRICULUM

To apply specifically our criterion of curriculum-making to the two reports under consideration; what are the major questions to which curriculum makers—either committees or individuals—must find the answers? Are they not these?

First: What are the abilities and the interests of the pupils?

Second: What materials (mathematical in this instance) do they need to study both as growing youth in school and as prospective citizens in a highly industrialized society?

Third: In what school grades shall these materials be utilized and in what order and arrangement within these grades?

Apparently simple and direct questions are they not? Textbook writers and makers of courses of study have not hesitated

to answer them categorically. In reality, however, there are no more puzzling questions in education today than these. For a decade scores of investigators have wracked their brains to find even tentative solutions. As a result a varied literature of objective analysis is growing up. As illustrations: the general mental abilities of pupils of various ages have been studied by objective methods; a host of statistical studies now supplies fairly definite evidence concerning the elimination and retardation of pupils in our schools; there has been much discussion, although little measurement, of children's interests and the dynamics of teaching.

It appears therefore that the background and illustrative techniques have been provided for the systematic and scientific study of the curriculum, mathematics included.

The two books before us make it possible to measure the extent to which these techniques are used in the reorganization of instruction in American secondary schools. The three major questions merely present the gross need. Their itemization will enable us to evaluate the two principal current methods of reorganization.

OBJECTIVE STUDIES ARE NEEDED ABOUT THE PUPILS, THEIR INTELLIGENCE, THEIR INTERESTS, THEIR FUTURE OCCUPATIONS AND THE LIKE

Whether he investigates it first or not, eventually the curriculum-maker must consider the abilities and interests of the pupils. Are there data available with which to answer questions about them? There are—very good data indeed. Did the National Committee and the Thorndike group make use of them or supply others? Professor Thorndike used some of them and has presented new facts. In its report, however, the National Committee considers the question only in a very limited and indirect way. That its recommendations were based on a detailed consideration of the facts developed in earlier studies of the psychology of childhood and adolescence in very doubtful.

Let us be more specific. The Thorndike report presents eight new objective studies which deal directly with the pupils who study mathematics: (1) changes in the quality of the pupils entering high school; (2) the occupations of high school graduates and non-graduates; (3) the intelligence of high school

pupils, including (a) a consideration of continuance in high school in relation to intelligence, (b) success in course in algebra in relation to intelligence; (4) the interests of pupils in algebra in comparison with other school subjects; (5) the interest of pupils in various features of algebraic learning; (6) individual and sex differences in algebraic ability and in interest in algebra; (7) the correlation between ability in algebra and general intellectual ability; (8) and the follow-up of that study in the analysis of special disability in algebra.

The difference in the method of the two groups—the subject-matter specialists and committees of teachers on the one hand, and the students of the scientific study of the curriculum on the other—is well illustrated by the manner in which they use these data concerning the pupils. The student of education feels that he must collate and study the known facts concerning the pupils who are to study the curriculum: how old they are, what their general and special abilities are, what interests teachers will have to combat or can make use of; what proportion of the pupils probably can deal successfully with the processes of the curriculum as predicted from the success of those who study the present ones, what changes will probably come about soon in the composition of the pupils and their abilities, what will probably be the future work of various groups of the student body and hence what probably should be the constitution of their curriculum.

These questions are definitely discussed by the Thorndike group. Data are tabulated upon which the answers to the questions rest and an elaborate interpretation of them is supplied. The National Committee's report, however, treats only two studies that bear on them and these incompletely and indirectly. The first was a study of the mathematical interests of high school pupils in which data on the following questions were collected from 6,978 pupils: Do pupils like or dislike mathematics? Why? What topics are liked, what topics disliked? Which high school subjects are preferred by pupils? The second study reported the facts collected by questionnaire by Dr. A. R. Crathorne from about 2,000 college freshmen as to whether they had then made a decision concerning what their life work would be and as to whether that decision had changed since they entered high school. The committee also collected and republished

several questionnaire studies of the opinions of lay citizens as to the value of high school mathematics.

The first point at issue then is this: the proper reorganization of the curriculum rests upon the detailed study of the facts of the pupil body. If these are not at hand then the curriculum-maker should collect, organize and interpret them, showing how they affect the materials of instruction. The professional student of the curriculum recognizes the value of the data and digs out of them facts that throw light upon his problems. Committees on the contrary tend to ignore them. There is no reason inherent in committee procedure that makes this course necessary except possibly the fact that committee personnel has never yet included the professional student of the curriculum.

OBJECTIVE STUDIES ARE NEEDED THAT DEAL WITH THE CONTENT OF THE CURRICULUM

The second major field for investigation by the curriculum-maker is the determination of what to teach. The National Committee, though not presenting convincing reasons therefore, does definitely recommend the topics to be taught in the mathematics courses of all grades from seven to twelve inclusive and prescribes certain courses to be required of all pupils in grades seven, eight, and nine. This is a courageous thing to do. Committees from the Committee of Ten down to date have done it. They regard that as their primary function. Yet it is safe to say that professional students of the curriculum would not undertake it in any high school subject. Why? The data upon which to make the recommendation are not at hand. The professional curriculum-maker insists that his first task is to discover social and psychological needs. It is certainly not his first function to recommend the topics for courses and the number of years the courses will require. The more he studies society and children the more hesitant he becomes to recommend the details of the curriculum.

To say that all boys and girls in a great nation should use valuable time in the mastery of the twenty-odd processes and operations of elementary algebra, for example, one would wish to be fairly sure either that they would be called upon to use the skill rather frequently or that the learning of it was necessary to enter into some mode of living, to grasp and use fundamental

generalizations or to appreciate and understand some contemporary problem. Can the curriculum-maker be sure that all or most pupils will actually use the operations of elementary algebra unless he has determined the use that human beings do make of them? Is it not the curriculum-maker's obligation to base his recommendations on the facts of human needs? If the facts have not been canvassed is not the curriculum-maker obligated to canvass them? Several inventories of the social uses of mathematics have already been made. These inventories have reported the use of algebra in other high school studies, the social uses of geometry, the algebraic operations used by workers in industry and in preparation for semi-professional positions in industry.

At this phase of their studies, as they did when dealing with the facts of the student body, the Thorndike group made investigations of the uses of algebra. They, too, make recommendations concerning the topics of algebra that they believe can be defended in a required curriculum. There is a significant difference, however, between their procedure and that of the committee: they collected certain facts of the uses of algebra first and based their recommendations upon those facts. It would have been my personal inclination to have secured more facts than they did. The chief point at issue, however, is well illustrated by what they did as contrasted with what the committee did. Recommendations concerning what to teach must be based on the facts of human needs, social and psychological. Is not the primary task that of finding the facts and interpreting them? Unless National Committees can be persuaded that that is their first duty will it not be difficult to justify their existence in American education?

On the topic of the "uses of algebra" the Thorndike report presents studies of: (1) the uses of algebra in other high school subjects; (2) the uses of algebra and geometry shown by an inventory of the Encyclopedia Brittanica; and (3) the uses of algebraic abilities as a preparation for work in the sciences in colleges and professional schools. The last study makes use of the data collected and reported by the National Committee itself. Mr. Thorndike extends the data, however, and makes a very much more complete treatment and interpretation of them than does the committee itself. In this study the National Com-

mittee collects the judgments of a very small body of college teachers, of the physical and social sciences, as to which operations of algebra, demonstrative geometry, trigonometry and analytic geometry were (a) essential, (b) of considerable value, (c) of some value, and (d) of no value in the study of their subjects in college. Students of the curriculum will appreciate the work of the committee in collecting these data.

Let us not forget, however, that they relate to the curriculum of that very small body of students who continue in school through college courses. These judgments help very little in settling the question of what to teach pupils in the junior high school. One does not find explicit reference, in the committee's report, to the earlier investigations which help in this matter. Nevertheless there can be little doubt that the members of the committee made themselves familiar with the contributions of various students of the curriculum in arithmetic and mathematics. In their forward-looking recommendations that all complicated mathematical operations be eliminated one sees the extent to which the committee cut itself loose from adherence to many of the hoary traditions of mathematics.

There were many unanswered questions as to the uses of particular processes of mathematics, some of which could have been answered if the committee had set up an objective analysis of social needs. This, in spite of its very large financial endowment, it did not do. Students of the curriculum will regret this greatly. I for one cannot bring myself to agree that it is a better use of funds to pay the expenses of college and high school teachers of mathematics to discuss in prolonged conferences the written reports of individual members than it is to endow objective studies of social and psychological needs. The round-table meetings do good and if funds are at hand for both those and the objective studies let us do both. In fact, once the fundamental studies are made, extended conference discussion of them is pre-eminently worth financing. But without the facts upon which the discussion is based how far can it proceed effectively? It can proceed only to the point of laying down the fundamental philosophy of curriculum-making in a democracy and of blocking out a definite program of investigation, experimentation and debate. Once that is accomplished the role of investigation begins. Round-table conference is fallen back upon periodi-

cally of course to check up the direction in which objective investigation is moving, the technique employed and the conclusions drawn en route. But it is to the collection of facts about society to which we have not yet sufficiently addressed ourselves.

It is all the more necessary that these objective analyses of society be made because of the halo that has grown up around the conventional courses. The general tone of the committee report itself—in spite of the fact that it was made by two "liberals"—exhibits the reverence in which teachers hold the subject matter that they teach. In sharp contrast to it is Professor Thorndike's much more objective analysis of the uses of algebra, the nature of the fundamental algebraic abilities, favorable and unfavorable modes of developing them and the amount of practice needed to learn particular operations.

The comment that there are many unsolved problems concerning the uses of mathematics reminds us that the Thorndike study itself by no means took care of all of them. The analysis of the uses of algebra in other high school subjects, for example, was made by his group much more completely than were the earlier ones and it settles for a decade, perhaps longer, that particular question. The question of what are the mathematical reading needs of people, however, could have been attacked directly, for example, by a study of the newspaper and magazine reading of the general public. Mr. Thorndike's associates tabulated the extent to which readers of articles in the Encyclopedia Brittanica need to know mathematical terms and operations. Readers of these articles, however, are typical of the same small proportion of the total public that uses mathematics in college courses (and most of them never again). Hence his study throws light primarily on the elective courses of the senior high school and college, not on the courses to be required of all.

Beyond these studies of the uses of mathematics the investigations of the committee and of the Thorndike group were not extended. From the Thorndike report one can obtain a direct interpretation of the data which deal with the topic of courses in algebra. Supporting its conclusions by evidence the Thorndike report analyses the extent to which the topics of conventional courses will be used. In this way it is possible to base curriculum recommendations on more than minority opinion. The criterion is objective and for each topic answers the ques-

tions: (1) Who will use it? (2) How much use will be made of it? Thus the professional student of the curriculum is not blinded by the halo of the past. He is single-track minded to the extent that *he recognizes only one great goal—namely, that of discovering what mathematical processes people need to take an intelligent part in the work and play of the world today. He postulates, at the start, both his and other people's inability to reach this goal from his own inner conscious processes. He believes that nothing short of an objective determination of people's activities will supply the details of the curriculum.*

OBJECTIVE STUDIES ARE NEEDED FOR MENTAL DISCIPLINE IN MATHEMATICAL INSTRUCTION

Perhaps the reader is impatient with the attention which has been paid to the "uses of mathematics." No doubt the members of the National Committee themselves will be of that mind for they pin their faith greatly to another criterion for teaching mathematics—namely, mental discipline.

For generations, mathematics, like the classics and science and history and other subjects, has been taught because its sponsors believed it "trained pupils to think." An enormous literature of opinion has been created to show that mathematics develops powers of logical thinking, that it brings about the ability to draw generalizations. Now, I personally think that the data of mathematics are peculiarly adapted to give pupils practice in drawing generalizations, that is to seeing relations between things which change together. But the more I study the data of the objective investigations of mental discipline, the more loath I am to choose subject-matter on the basis of a faith in "transfer."

The major curriculum question before us is not: Does training transfer? Of course it transfers. Thirty-five experimental studies provide the facts which one needs to form a judgment about that. The committee itself has made the data conveniently available to teachers of mathematics. It has reprinted my detailed summary of the 29 studies which were published prior to 1915, and has also presented Miss Blair's excellent extension of that work, thus bringing the data of the controversy over "transfer of training" completely up to date. Students of the problem will appreciate this work. Miss Blair also collected

the judgments of psychologists who have worked over the data of transfer. Summarized, these judgments say with varying emphases what I just said, "of course there is transfer."

But, to repeat, that is not the major curriculum question. Students of the curriculum face three other questions of discipline that are much more insistent than the academic one of training transfer. These questions are: First, does the study of high school mathematics as now constituted increase one's ability to think? Second, if it does not, can high school mathematics be so organized that it will? Third, and most important of all, is the increase in ability to think, which comes (or can come) from the study of mathematics great enough to require pupils to devote equal or more time to it than they do to the problems of physical science, natural science, social science, literature and other esthetic subjects?

Now can any "committee" sit in judgment on these questions? It cannot, and for one very good reason: the necessary facts are not at hand or at least were not in 1922. Will it not be granted that these three questions are fundamental and that they must be answered (and for each department) before we can make headway in the permanent reconstruction of the school curriculum? How can they be answered? Certainly not by collecting the opinions of psychologists as to whether training transfers! On the contrary the first two questions can be answered by testing large numbers of children before and after one year, two years, three years or more years of instruction in mathematics. It is not asking too much of well-financed National Committees in these days to expect them to do this. The tests are available and the technique of testing and statistical treatment is part and parcel of the equipment of curriculum specialists.

The National Committee did not attempt to find the answer to these questions. Mr. Thorndike did, although not as a part of his specific investigations of algebra. He did it as a separate inquiry. He is presenting his data in the current (January) issue and the following issue of the Journal of Educational Psychology. He shows first, that one year's study of either algebra or geometry *as now organized* does increase one's ability to think—*by a small amount;* second, that the gain is no larger than that due to the study of other subjects, *as now organized.*

Startlingly enough—bookkeeping, biology, cooking and sewing increase one's ability to think as much—even more in some instances—than does the study of mathematics or the classics!

In this present writing I am not primarily concerned with the *findings* of these investigations. *It is on the methods employed by those who are engaged in reorganizing the curriculum that we are focusing our thought.* Dr. Thorndike's success in answering the first of our three questions is an impressive exhibit of the fact that the committee could have carried on objective studies of disciplinary values. To do it, however, their personnel would have had to be enlarged to include curriculum specialists. Is it not becoming clearer and clearer that the personnel of National Committees must include curriculum specialists? The National Committee could have set up important studies like that referred to, which by this time would have supplied the answer to the second of our major questions of the disciplinary values of mathematics. The third question is much more difficult. We do not have the facts with which to answer it. They will be expensive to discover and difficult to interpret. Committees with funds to dispense can contribute greatly to the discovery of the facts by setting up extensive evaluations of both the social and disciplinary values of the major divisions of the school curriculum. Is the evidence not convincing that these evaluations must be objective?

OBJECTIVE STUDIES ARE NEEDED CONCERNING PROBLEMS OF EXPERIMENTAL TEACHING

There remains to be considered one of the most difficult phases of curriculum construction. I refer to the third of our major questions: Paraphrased it reads: How shall mathematics (or any other "subject-matter") be taught? A host of puzzling queries confronts us. Some deal with the problem of consignment of materials and activities to the grades of the school. Others with their teaching arrangement. Here are three illustrations: First, where shall the "introduction to algebra" be given? Shall the pupil first meet it in grade IX as he has traditionally or meet it in grade VII as certain new textbooks organize it, or even perhaps in grade VI? Second, in what school grades and through what graduation shall the notion of relationship be built up, the concept of correlation, functionality,

dependence—call it what you will? Third, shall "signed numbers" be taught early or late in our elementary algebra instruction?

These are typical of the most difficult psychological questions which the curriculum-maker is called upon to answer. Are facts available with which to answer them? To answer some of them—yes; for most of them—no. A generation of teaching experience—sure but wasteful—has taught us that certain "subjects" like algebra or parts of subjects can be learned in the ninth, or eighth, or even in the seventh grades, by pupils of particular degrees of verbal intelligence. To this day, however, there has been no systematic, controlled, and measured experimentation designed deliberately to solve these problems of grade placement.

For each topic or item the major problem can be put simply: What is the earliest school grade in which this principle or process can be learned effectively the learning of which at the same time will fit into the total graded scheme of instruction in that subject? This problem can be attacked directly and experimentally. All that is required is to teach the same operations to a large number of pupils in several adjacent grades, say VI, VII, VIII and IX if tradition has already placed the material in say grade IX. The facts of the pupils' intelligence and their attainments can be determined objectively. That the experiment is difficult is not a sufficient reason for neglecting it. Its importance is paramount. The science of education has developed a technique which is adequate for its solution. It is becoming apparent that our National Committees can command the financial resources to carry through such experiments. Is it not imperative that these national bodies organize the machinery of such experimentation?

I hasten to comment on the procedure of the National Committee on Mathematical Requirements. They did not set up, or endow grade placement and teaching experiments. That will be regretted I am confident. They could have sponsored such experiments. To have done so would have greatly furthered the development of exact curriculum-construction in mathematics.

It is greatly to their credit, however, that they made no definite recommendations for grade placement. They recommend

the topics to govern instructional units but decline to advise as to teaching arrangement. The latter position is taken on the ground that the basic facts are not known. Their recommendations, optional in character, are permeated distinctly by an experimental attitude and should help create such an attitude on the part of mathematics teachers.

The Thorndike group likewise neglected *experimental* problems. The solution of problems of grade placement and teaching arrangement of materials and activities was not undertaken. Very likely they could not be attacked with the resources at the group's command. Teaching experiments, carried out on a scale large enough to supply substantial proof will be very expensive, will occupy the energies of a considerable body of trained and experienced experimentalists in the classroom and will extend over several years of continuous effort—probably not less than five.

As an important phase of this vast problem of experimental teaching we are confronted by the difficult task of interpreting and apply what we know about the psychology of learning to the analysis of learning. The National Committee, very properly, no doubt, does not undertake the task. The task is one for educational psychologists. The problem is the complicated one of detecting and describing the nature of the abilities called into play in learning mathematics. There are two ways to attack the problem. One is the method of experimental analysis and measurement. The other applies directly current psychological thought to the analysis of the abilities involved.

Both the Thorndike group and the National Committee describe excellently the tests that are now available for the use of teachers of mathematics. Professor Upton's very complete chapter is an excellent exposition and evaluation of the tests. But the committee has not used the tests to evaluate results obtained from present instruction nor to analyze the constituent abilities of mathematical learning.

Again it is the students of education who did it. The Thorndike group not only presents new tests and evaluates old ones—it also evaluates results obtained from present instruction. One of the chief contributions of the study is Mr. Thorndike's analysis of the abilities called into play in algebra. He has followed the same procedure for algebraic learning that he de-

veloped in his *Psychology of Arithmetic*. The procedure first states what the abilities are that need to be developed; for example the ability to understand and frame formulas, ability with equations (including an analysis of the psychology of the equation), ability with problems (including the psychology of problem solving), and ability with graphs. Next the procedure states what contribution the current dynamic psychology can make to the perfection of habits involved in learning algebraic operations. It considers from a thoroughly inductive point of view such questions as: provisions for habits now neglected, the elimination of unnecessary habits, adaptable habits vs. flexible rules, learning meanings, learning algebraic computation, learning general principles, the selection of the mental connections to be learned and other related matters.

In the present undeveloped state of our science it is most important to have these analyses of the abilities which are utilized in learning our school materials. No more conspicuous illustration can be found in the two reports under discussion of the fact that National Committees must include in their personnel specialists in the study of the curriculum. Committees are obligated to discuss problems of teaching; hence problems of learning come within their purview.

More important, however, than the analysis of abilities is the discovery of how the abilities are developed. The committee does not discuss the problem. Mr. Thorndike devotes 160 pages to it. Probably his chief contribution at this point is his analysis of the practice now given to the learning of algebraic operations and his study of the psychology of drill. The former is objective, and includes a count of the opportunities for practice offered in current school textbooks. The latter analyzes teachers' inability to estimate how much practice should be given on particular operations and presents a new estimate of the amounts of practice that should be given.

THE CRUCIAL NEED FOR CONTROLLED EXPERIMENTATION IN AMERICAN EDUCATION

These two studies of secondary mathematics bring out in sharp relief one of the most patent gaps in the development of the scientific study of education. Not a single *experimental* study is reported, either by the National Committee or by the

Thorndike group. It is true that the committee prints a 100-page description by R. Schorling, of the way mathematics is taught in fifteen different schools. This is called "Experimental Courses in Secondary School Mathematics." The unit is improperly named, however, for only one of the enterprises discussed could be thought of as "experimental" (as the writer himself points out) and the description of that one is totally inadequate. This omission is not peculiar to these two studies under discussion—it is typical of the whole scientific movement in education.

The great foundations are doing a work of incalculable value by making possible the inventory of our school practices and, to some extent, the objective analysis of them. But there is a dire need that they pay more attention to the subsidizing of *experimental* education. Mr. Thorndike's studies illustrate one thing they are now doing. They are endowing laboratory analyses of existing conditions and the direct application of current psychological knowledge to the improvement of teaching. But this type of work, fundamentally important as it is, represents but one phase of the scientific study of education.

Is there, in America to day, one scientifically controlled classroom experiment in mathematics? It is my confident judgment that there is not. Yet there are more than fifty public school systems in each of which there is in working order a "bureau of research," directed by a person trained in the technique of measurement and in most cases of experimentation. It is practically impossible for an editor to discover for journal publication controlled classroom experimentation. Furthermore, it is practically impossible to persuade graduate students in our schools of education to undertake dissertation problems in experimentation. Why? Primarily because it costs too much; it takes two or three years to complete one controlled experiment. Our major "research" energies are still devoted to giving tests and to presenting "norms."

The way out of our dilemma is clear. The educational foundations are spending large amounts of money on committee procedure and on laboratory analyses of educational processes. *The mathematics report is by far the best report that has came from such endowment. Should it not be the last endowed committee report in American education, however, that does not*

carry on scientific studies of the abilities of children, of the needs of society and of the learning process?

In conclusion may I plead for the financing by our great foundations of a more complete and scientific program of procedure for National Committees. It is clear that these committees can render great aid in the improvement of school processes. Permanent reconstruction will be difficult without their services. Can they not, however, be so constituted that their programs will be complete and will employ scientific technique?

The work of the National Committees, with large financial support, is about to be launched. What will be the procedure of these committees?

To further the critical discussion of the matter I reprint in part an earlier proposal.

A PROGRAM FOR NATIONAL COMMITTEES

A committee should act in three capacities if it wishes to improve school practice generally. First, it should act as a deliberative body of specialists, stating educational aims and criteria; second, it should organize important investigations of social and psychological needs which underlie the curriculum; third, it should act as a national clearing-house and forum for controversial discussion.

A. THE COMMITTEE AS A DELIBERATIVE BODY OF SPECIALISTS STATING AIMS AND CRITERIA

The most important task of the committee is to draw up a scientific program for curriculum-making. This will necessitate frequent and prolonged round-table conferences.

The program should include, first, a definite statement of aims and outcomes of instruction; second, criteria for the inclusion of material in the courses, for assigning materials to school grades, and for presenting materials within grades. Such statements can be made only by getting the best composite judgment of trained specialists—a judgment which is the result of mature deliberation and round-table discussions. Two types of specialists should contribute to this discussion: (1) those trained in the study of the validity of materials; (2) those trained in

the science of curriculum-making and educational psychology. It is inconceivable that a curriculum can be made properly by either group working alone.

B. THE COMMITTEE AS AN ORGANIZER OF INVESTIGATIONS OF SOCIAL AND PSYCHOLOGICAL NEEDS

The program of a national committee, in the second place, should be investigational. Once the fundamental principles are established, detailed studies of human needs and results must be made. Five types of work appear to be important.

1. Determine the exact status of the present teaching in our public schools. This means an accurate compilation and interpretation of current courses of study from an adequate number of communities—certainly several hundred—selected so as to make a perfectly representative sample of the whole country on such matters as: (a) aims, objectives, and specified outcomes of instruction; (b) scope of the course which is covered in each grade; (c) time devoted to the subjects of each grade; (d) the precise materials used by the pupils—textbooks, activities employed, miscellaneous materials; (e) by the use of trained specialists, adequately financed, analyze by careful quantitative methods and interpret thoroughly the content of school textbooks and other materials and also class activities, actually in operation in public schools. The purpose of this is to determine precisely the content and arrangement of our present curriculum. We must know definitely what present practice is in order constructively to evaluate it and scientifically to reconstruct it.

2. Evaluate critically and constructively the scientific investigations of curriculum-making in the subject under consideration that have already been published. In like fashion, borrow and make similar use of unpublished curriculum studies in our universities and schools of education. The findings of many such investigations are deposited in the libraries of universities like the University of Iowa, University of Chicago, Teachers College, Leland Stanford Junior University, and others.

3. Make and publish an exhaustive evaluation of representative examples of experimental courses. A score or more of important innovations can always be found among the thousands

of schools and school systems. Scrutiny and careful weighing of the purposes, programs, and accomplishments of these new types of courses, some of which are truly experimental, are the most important functions of national committees.

4. Test objectively the results obtained from present instruction in a sampling of typical public schools by the employment of standard measures of attainment. This would reveal to what extent pupils have mastered great relationships, principles, and "laws"; to what extent they have developed a command of technique and have learned how to use it.

5. Make scientific investigations of social and psychological needs for the purpose of discovering what should be included in the courses, to what grades different materials should be assigned, and in what order and arrangement they should be presented. Such investigations should be delegated to expert collaborators—specialists trained in educational research. Similarly, the committee should endeavor to stimulate constructive research by others for the discovery of what ought to be taught and in what grades the material should be given.

C. THE COMMITTEE AS A CLEARING HOUSE AND FORUM

It should stimulate controversial discussion by publishing and debating with school people generally its own programs, various other programs of work, different hypotheses concerning the selection and arrangement of subject-matter, theories of teaching, principles of organization, and the like. It should direct this discussion to the consideration of big issues and important questions so that after some years both clear thinking and experimentation will make possible a widespread agreement on fundamental matters. It should get official sanction for, and give widespread publicity to, the findings obtained from the committee's careful investigations, deliberations, and decisions.

D. THE CONTINUING COMMITTEE

As a continuing or standing body, once its recommendations are put into practice, it should study their application in public schools; collect reactions of school people to the proposed program; test results obtained by different organizations of material found by the committees to be socially valuable; finally, report these findings periodically to the school public.

PRESENT TENDENCIES IN HIGH SCHOOL MATHEMATICS[1]

By JONATHAN T. RORER
William Penn High School, Philadelphia

The present tendencies in high school mathematics are well reflected by the report of the National Committee on Mathematical Requirements, which was organized in 1916, and which has completed the most exhaustive research ever attempted into the conditions now existing in mathematics teaching. The work of the National Committee already has had a far-reaching influence on mathematics teaching. It represents a great piece of cooperative work, a systematic presentation of the thought and practice of many members of this association and of other teachers interested in the improvement of mathematics teaching. It will be for a long while a standard source of information for anyone who desires to know modern tendencies in secondary school mathematics.

Mathematics teaching, like other forms of teaching, is a function of at least four variables—the teacher; the subject matter, or course; the teaching instruments, or equipment; and the pupil.

It is easily seen by anyone of experience that great changes have taken place in our teaching during the past decade. It is the purpose of this paper briefly to review these changes and tendencies and to discuss them in the light of present educational theory.

1. The Teacher. We have today better equipped teachers than were in the schools of yesterday. The young teachers frequently are graduates of a school of education connected with one of our universities. They have had a broad educational training combined with specialization in their chosen field. They look upon education as a profession and are less likely to use teaching as a stepping-stone to law, medicine, or another profession. They are a happy mean between the old time normal school graduate, who was apt to be infected with a useless outworn pedagogy, and the abnormal collegiate book worm whose interest was too narrowly academic.

[1] A paper read before the Association of Teachers of Mathematics in the Middle States and Maryland at the Bethlehem meeting, December 1, 1923.

University courses on methods of teaching mathematics are beginning to bear fruit. In this way a young teacher receives in one or two courses many ideas which otherwise would require years of active experience. He starts to compare methods, to analyze his teaching results, to be watchful for ideas that will make his teaching more effective. He becomes acquainted with the best instruments of teaching, with good textbooks and tests. He is introduced to the literature of his chosen profession. Formerly our teachers were those supposed to be possessed of a superior knowledge of mathematics. The new type should know as much mathmatics as the older type, and *more*, if emphasis is placed on real knowledge, but in addition to thorough assimilation of subject matter, our modern teacher recognizes that he must strive for proficiency in teaching—that subtle process which inspires the pupil to learn and which guides him into logical, economical, and dynamic thought processes.

Correspondence, extra-mural, and summer school courses also have greatly helped hundreds of teachers of many years' experience. The teacher is now rare who has not come into contact and into sympathy with improved ideas in the teaching of mathematics through books on methods, or through college or university courses, or through meetings of their fellow teachers such as this.

The teacher of today, too, differs, from the older type of teacher in his attitude toward the pupil. There is a very definite purpose to understand the pupil as well as the subject matter, to get his point of view, to appreciate his likes and dislikes in order that his failures and his successes may be properly diagnosed.

II. The Subject Matter, or the Course. We are all aware of the changes in the subject matter of mathematics teaching. The junior high school has hastened the introduction of general mathematics into the seventh and eighth grades. The old time continuous review of arithmetic with the addition of technical commercial problems is rapidly passing away. Algebraic notation and some formal algebra have established themselves in the seventh and eighth grades. The former mensuration has been enriched with compass and protractor constructions. Common sense in the number of decimals to be used, estimation of re-

sults, and a wiser choice of material are among the improvements.

Many senior high schools are also offering general mathematics in the ninth grade as an alternate to the usual algebra. As the colleges have not yet given the unit credit to general mathematics that they give to algebra, it is necessary to continue the algebra course for all college preparatory students. These students must consider carefully the available courses with a view of securing the fifteen required units in their specified distribution. Whether they take algebra or general mathematics in the ninth grade, the comprehensive examination of the college board is well within their scope, if they continue with second year algebra and with geometry. But until the unit is granted by the colleges without question, such candidates will naturally not be given general mathematics.

The private preparatory school has a much simpler problem than the public high school in facing the decision between algebra and general mathematics. It is clear sailing under the old method; therefore, the introduction of general mathematics is somewhat doubtful.

With the public high school the elective principle is more generally recognized. The requirement in mathematics is but one year in very many schools in this territory. As a consequence, a large percentage do not take the elective courses. Here we have an unanswerable argument for general mathematics in the ninth grade. Knowing that these pupils are with us for one year only, are we justified in confining the course to algebra? Many of us think that we should give such pupils a more general course. But it is objected that we are not responsible for their decision to curtail their time to but one year of mathematics. This is true, yet we reply that we are more likely to retain such pupils for subsequent years of mathematics by a varied appeal in the ninth grade than by the one appeal of algebra.

The discussion of the merits and the weaknesses of composite or general mathematics will come more naturally under the second paper of this program than under the first. But it must be observed in passing, that the general mathematics movement is growing rather than waning. It is here to stay. Our part should be constructive—to help assemble those most funda-

mental parts of mathematics which have manifold correlations with common life situations, and to study in what order these parts should be placed to produce the best course of study.

Those who doubt the increased popularity of the composite course should observe the large number of composite textbooks issued during the last few years, not only in the high school field, but also in the technical school and college fields. Miss Vera Sanford, of the Lincoln School (MATHEMATICS TEACHER, XVI, 4, pp. 206-214), has analyzed eleven textbooks of the unified type used in some colleges during the freshman year. She states that her list is a partial one. To it may be added as many more excellent composite books suitable for colleges and technical schools. It is safe to say that a complete list of these advanced composites would contain over twenty-five titles.

The College Entrance Board (Document 107, May 15, 1923) has in a rather veiled manner endorsed composite mathematics in their new definition of elementary algebra. The new definition briefly stated as to topics is:

1. Simple formulas.
2. Graphs and their interpretation.
3. Negative numbers.
4. Linear equations in one and two unknowns.
5. Ratio, proportion, and variation.
6. Essentials of algebraic technique: Fundamental operations, factoring and fractions; verification.
7. Exponents and radicals.
8. Numerical trigonometry.

The "Notes to Teachers," which are appended to the minimum outline of the course, to which reference has just been made, state: ". . . A less extensive treatment of certain purely algebraic topics than has hitherto been the practice in elementary algebra is herewith specified in order that time may be saved for the introduction of intuitive geometry and numerical trigonometry, as well as for better emphasis on the great basal principles of algebra."

A comparison of the former requirements (Document 105, December 1, 1922) with the new shows the omission in the new of these topics:

1. Highest Common Factor and Least Common Multiple by factoring. Inspection replaces factoring.

2. Square root of a polynomial.

3. Negative exponents.

You will find the insertion of these ideas:

1. The graph and its interpretation, including the type, $y = x^2 - 2x + 3$.

2. Variation in addition to ratio and proportion. (The Board says it is not necessary to treat ratio and proportion as a distinct topic. Yet this is the very thing the Board does in its own outline of topics, and with the inclusion of the idea of variation!)

3. Numerical trigonometry.

When the teacher balances his losses and gains, he can hardly see where the saving of time for the intuitive geometry is to come from. Although it is frequently stated that exercises are to be confined to simple cases, yet on the other hand, the thorough understanding of principles is so emphasized, that teachers will feel that the requirements are indefinite and in consequence the amount of drill required for the new course may exceed that of the old.

The changes that will be brought about in geometry by the new requirements of the College Entrance Board (Document 108) are too numerous and complex to be adequately discussed in this paper.

Just a few comments must be made. In plane geometry the syllabus contains 89 theorems and 20 constructions, of which but 31 theorems are starred, to be reproduced by the pupil. This will mean that in many schools, the other 59 theorems will be read "by title" only. The selection of these starred theorems is not always wise. What knowledge of the usual method of deductive reasoning can the superposition congruence theorems for triangles give? Yet these propositions are starred. These proofs are mainly descriptive—an exercise for an English teacher. They form the strongest temptation for parrot-like memory work. The inclusion of one of these propositions among the Hilbert postulates is another reason for not requiring the proof.

The new requirement in solid geometry is 92 theorems; the number starred is but 22, so distributed that these topics are not represented—loci, polyhedral angles, cylinders, similar solids, and the measurement of the sphere. In order that the

attitude of the Board may be more clearly understood, I shall quote as follows from their syllabi:

"In the case of the unstarred propositions, the candidate will be expected to be familiar with their content, so as to be able to answer questions about their substance or use them as a basis for solving originals, but he will not be assumed to have their demonstrations in mind."

The Board has created a new one unit course in plane and solid geometry, which is called a minor requirement as compared with the requirement covering the two separate courses. The new course includes 96 theorems and 13 constructions, of which 33 are starred. It appears that solid geometry has been crowded out of many schools. The new minor requirement course will enable such schools to give a one-unit course that will cover the essentials of both subjects. It will be interesting to see how this will work out in practice.

The whole reform movement which is based on ideas that have my entire sympathy faces the dangerous tendency of placing before the public more ideas than can be assimilated, in the time allowed. Unless we are guarded, we shall increase the superficiality which our foreign critics claim to be characteristic of American education.

III. The Equipment. Our consideration of the school equipment must be very brief. Let us notice the textbooks first. When many of us started teaching, the number of textbooks was very few. The stars of Davies and Loomis were setting; Newcomb, comet-like, was visible for a little while; Brooks, too, had rendered a good service, more especially in arithmetic. The great sun in the firmament was Wentworth, and it is safe to say that his name has been placed on more textbooks than that of any other teacher, unless we except that of Euclid.

What a contrast today! Textbook writers are as numerous as newspaper cartoonists, and many of them are just about as useless. The new textbooks are not sufficiently unique; they are too much alike. Fortunately, on the whole, the new books are attractive and the subject matter is well chosen. They generally adapt themselves to modern ideas. Manipulation of symbols, as such, is reduced to a minimum; the automobile and the airplane have replaced the couriers and the hare and hounds; A and B, so delightfully ridiculed by Stephen Leacock, when they worked

two or three days, have given way to Alden and Alice, Lester and Loraine, though John, James and Mary are still very popular. It is noticeable that these young people no longer do a piece of work in five days, or even in five years, for working is not quite as fashionable as it once was.

Problem material, up-to-date, and in the language of the child's environment is furnished by the author who has brought the "hot dog" into problems. "How many 'doggies' were sold? How many pounds?" Notice the vocational guidance when he calmly states as a preamble to the problems—"One of the most profitable stands at a fair is the small 'doggie,' or 'hot-dog' stand. The money necessary to start such a stand is small in amount, the danger of losing is not large if the business is operated in a careful manner, and there is a lot of fun mixed in with the hard work."

The mention of the canine or bovine cylinders brings to my mind an awful example of pedagogical advice which a suggestive book on directing study has recently given us. This problem, quite ordinary and in place, is proposed for sixth or seventh grade:

"If four dogs cost \$100, what will five dogs cost?" The work is expressed thus:

$$\frac{4}{\text{dogs}} = \$100$$

$$\frac{1}{\text{dog}} = \frac{1}{4} \text{ of } \$100 = \$25$$

$$\frac{5}{\text{dogs}} = 5 \times \$25 = \$125$$

The author has invented this notation, believing it to be analogous to fractional parts, for he proceeds with the problem:

"If four-fifths of my money is \$100, how much money have I?" This he expresses:

$$\frac{4}{\text{fifths}} \text{ (of my m.)} = \$100$$

Etc. etc. etc.

He is oblivious to the fact that he has now inserted "(of my m.)" in the new equation, entirely absent from the dog problem.

Next he exclaims, "Above all see to it that *dogs* and fifths are denominators." His sincerity crops out when he writes in parenthesis, "(There will be need of clarifying the idea of unity.)"!

Leacock still has material for his essays, even if A and B are out of work!

Next we must note the tendency in the schools to have a real mathematical equipment. We are getting beyond the minimum necessities, pencil and paper, or chalk and blackboard. The compass, protractor, the transit, the sextant, models of various kinds, slide rules, tables, calculating machines, are now more generally with us.

A part of the necessary equipment is the standard test. The success in the field of algebra and arithmetic, on the whole, is satisfactory, but geometry is still virgin soil. The few who have labored in this field, have worked intelligently, but with results unsatisfactory even to themselves. Trigonometry and more advanced mathematics have not as yet to my knowledge been attacked by the standard test makers.

IV. The Pupil. The changed conditions in our schools are all traceable to the modern type of pupil. Formerly we taught algebra and geometry to the selected few; now we teach everybody. The best of today are equal to or better than the best of yesterday, but the average pupil is not nearly so bright and surely not so industrious as the pupil of former days. The great predominance of mediocre teaching material has been a retarding influence on the progress of the brighter pupils. The books, the courses of study, the classroom teaching have all been adjusted to this great army mobilized by compulsory education laws. The better pupils have undoubtedly suffered and "many pearls have been cast before swine." We are beginning to emerge from the follies produced by these swarms of the uneducatible, as Dr. Lightner Witmer, the well known psychologist, calls them. Our schools are becoming more diversified in purpose. Industrial and vocational education, under proper guidance, will relieve academic courses of many misfits. We shall also learn that industry itself can be educational, and the boy or girl who will do nothing in school will be allowed to do something in industry.

We are also taking care of the bright pupil. We are beginning to realize that he has his rights as well as the defective. There is no reason why he should be retarded by the subnormal, or even the mere normal. Classes and schools for gifted children are meeting this need.

These are some of the indications that the pupil is in reality the main topic of study of today.

On the whole, this cursory review shows that modern tendencies in mathematics teaching are encouraging, that the teachers are better prepared for their work, that the textbooks and equipment are very much superior to those of former days, that the courses are creating a live interest on the part of the pupils, and that the pupils' capacities, aims, and intellectual outlook are being made the objects of scientific study by our teachers in a way that is characteristic of modern educational progress.

CONTINUITY IN MATHEMATICS AND EVERYDAY LIFE

By W. V. LOVITT
Colorado College, Colorado Springs, Colo.

In Wentworth's plane geometry, page 107, we read: "By making the distinction between quantities measured in opposite directions, a theorem may often be so stated as to include two or more particular theorems."

The following theorem furnishes a good illustration:

The angle included between two lines of unlimited length that cut or touch a circumference is measured by half the sum of the intercepted arcs.

Here the word sum means the algebraic sum and includes both the arithmetical sum and the arithmetical difference of two quantities.

1. If the lines intersect at the center, the two intercepted arcs are equal, and half the sum will be one of the arcs. A central angle is measured by its intercepted arc.

2. If the lines intersect between the center and the circumference, the angle is measured by half the sum of the arcs.

3. If the lines intersect on the circumference, one of the arcs becomes zero and we have an inscribed angle or an angle formed by a tangent and a chord. In each case the angle is measured by half the sum of the intercepted arcs.

4. If the lines intersect without the circumference, then one of the arcs is negative and the algebraic sum is the arithmetical difference of the included arcs.

When the reasoning employed to prove a theorem is continued in the manner just illustrated to include two or more theorems, we are said to reason by the principle of continuity.

A small change in the coefficients of a quadratic equation produces a smal change in the roots. By continued small changes in the coefficients the roots of a quadratic may be made to vary by small changes from real and unequal, to real and equal, and then imaginary.

The student of mathematics will call to mind other illustrations of this principle of continuity.

That the common people have some conception of this principle is evidenced by a number of phrases, such as: treat all

alike; show no partiality; be consistent; do not blow hot then cold; do not make fish of one and flesh of another; what is sauce for the goose is sauce for the gander.

". . . the Colonel's Lady and Judy O'Grady
Are sisters under their skins!"

A recognition of the principle of continuity is nothing more than the recognition of a common principle underlying what at first seems to be isolated facts. It is in the nature of a generalization. The function idea is present. The dependence being such, that as one variable changes, the value of the function, *i. e.*, the other variable, changes and changes continuously. The demand for an article remaining about constant, as the supply increases the price decreases.

Different people react differently to the same stimulus.

"What did the Colonel's Lady think?
Nobody ever knew.
Somebody asked the Sergeant's wife,
An' she told 'em true!"

With this seeming difference in reaction, there is, however, fundamentally the same reaction, the difference is in the outward manifestation. Human conduct is much the same the world over.

". . . the things you will learn from the Yellow and Brown,
They'll 'elp you a lot with the White."

In geometry we find three separate theorems:

1. In any triangle, the square on a side opposite an acute angle is equal to the sum of the squares on the other two sides diminished by twice the rectangle contained by one of those sides and the projection upon it of the other side.

2. The square on the hypotenuse of a right angled triangle is equal to the sum of the squares upon the two sides.

3. In any triangle, the square on a side opposite an obtuse angle is equal to the sum of the squares on the other two sides increased by twice the rectangle contained by one of those sides and the projection upon it of the other side.

In trigonometry these three theorems are covered by one statement:

In any triangle, the square on any side is equal to the sum of the squares on the other two sides diminished by twice the product of those two sides times the cosine of the angle included between those sides.

In analytic geometry the single equation

$$Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + C = 0$$

can by a continuous change of parameters be made to represent a point, a circle, an ellipse, a parabola, an hyperbola, two intersecting straight lines, two parallel straight lines, two coincident straight lines.

The area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is πab.

Let us hold a fast and let b vary continuously until it becomes equal to a. The area of the ellipse changes continuously but always remains equal to πab. In the limiting case $b = a$ and the area becomes πa^2 while the ellipse becomes a circle.

Here the result for the limiting or degenerate case is obtained by the principle of continuity from a knowledge of what happens in the non-degenerate case. In general this principle of continuity is used to find out what happens in the limiting and degenerate cases when we have a knowledge of the non-degenerate case.

We have the general theorem: two cubics intersect in nine points. Any cubic through eight of these points will pass through the ninth point.

By a continuous change of parameters a cubic may be composed of a conic and a straight line. Then the theorem would read: two cubics intersect in nine points. If six of these points are on a conic then a straight line through two of the remaining points will pass through the ninth point.

By a further continuous change of parameters a cubic may break up into three linear factors represented by three straight lines. Now the theorem reads: two sets of three lines intersect in nine points. If six of these points are on a conic, then a straight line through two of the remaining points will pass through the ninth point.

If our six original lines, forming two cubics, are so situated, with respect to the conic, that they form an inscribed pentagon and a tangent to the conic at one of the vertices of the pentagon, then our theorem becomes Pascal's theorem: in any pentagon inscribed in a conic, the points of intersection of two pairs of non-adjacent sides lie in a straight line with that point in which the fifth side is cut by the tangent at the opposite vertex.

In ordinary life the principle of continuity is present in our ideas of growth. Strong oaks from little acorns grow.

> Vice is a monster of such dreadful mien,
> That to be hated, needs but to be seen.
> Yet seen too oft, familiar with its face,
> We first, endure, then pity, then embrace.

In biology, the principle of continuity is expressed in the exposition of the theory of evolution. The present forms of life have been evolved through the countless ages, by a continuous change, from more primitive types.

In short, we find that this principle of continuity is one of general applicability. It is present, in varied form, in many of our non-mathematical activities.

ORAL WORK IN HIGH SCHOOL ALGEBRA[1]

By MISS S. HELEN TAYLOR
University High School, Urbana, Ill.

Definition: By oral work I refer to all work of which none is written, whether it is actually rehearsed in words or simply performed in thoughts. I shall include all class discussion under oral work, so that I may include what I consider a very valuable part of oral work, the discussion of written work. I qualify this definition by remarking: first, that if the algebraic material is well organized, it is difficult to set off one section as oral and one as written, even for a given pupil; second, work which is suitable for oral work for one pupil is not necessarily so for another, and third, that in practice one passes over the boundary from oral to written work with great frequency and that this is desirable.

Basis of This Paper.—The algebra text in use in the University High School is Rietz, Crathorne, and Taylor. The first year course by these authors, though excellent from the standpoint of simplicity of principle and logical organization of material, has not a large number of problems at any point, so that it is necessary to introduce considerable supplementary material to meet the needs of an average class. Partly because of this and partly because we were particularly interested in trying some oral work, we, last year, introduced the text "Oral Algebra" by Golden and Fenno, edited by Mr. Meyers. From the use of this supplementary text we have become interested in the use of oral work in algebra in general. I feel that we have not seen the possibilities in any complete sense, nor have we used the text long enough, to state fully or positively the results of using oral algebra. But I shall attempt to organize and interpret some results we have had in this work.

Oral Work in Current Texts. In considering the place of oral work in high school algebra, we may first direct our attention to an examination of certain well known texts of today to discover how much emphasis is placed on oral work. For our purpose I have chosen the method of first reading the preface for a mention of oral work, then looking through the problem lists for

[1] Read before the Mathematics Section of High School Conference, Urbana, Ill., November 23, 1923.

marked "oral," 'and finally if none were so marked, I looked over the lists carefully to see how many might be used for oral work. The texts I examined do not in any way form an inclusive list. Each one was chosen either because it has a wide use or because it is a recently published book.

Hawkes, Luby and Touton. In the preface there is no mention of oral work. There are only two lists in the text marked "oral," one in the chapter on multiplication and one in the chapter on division. However, before each set of word problems in the first ten chapters, there is a set of translation exercises, which are best used for oral work. There is such an abundance of problems in the book that it is possible to adapt many to oral work.

Slaught and Lennes. The authors, in the preface, call attention to the large number of "exercises, oral and written." On examination of the material we find a very large number of oral exercises.

Wells and Hart. There is no mention of oral work in the preface nor are any of the exercises so marked. The material is sufficient in amount for some of the simpler problems to be used orally.

Wentworth and Smith. The authors do not refer to oral work but in every list a certain number at the first of the list are marked *oral*, and the remainder marked *written*. About one-third of all the problems in the text are marked *oral*.

Rietz, Crathorne and Taylor. These authors have no reference to oral work in the preface. There are no problem lists especially designated as oral and there is not a sufficient number of problems for oral use.

Breslich, Book I. Oral work is not considered in the preface. There are a number of lists in which the suggestion to do as much as possible mentally is given. There are a number of translation lists which may be called oral, though they are not so listed.

Ford and Ammerman. There is again no mention of oral work in the preface. The exercises are marked oral and written. In the first five chapters the oral work exceeds the written in amount.

Durell and Arnold. In the preface the authors refer to the large number of oral exercises in the use of algebraic language. Each exercise in written problems is preceded by an oral exercise in the use of algebraic language. Besides these lists there are many others marked "oral."

Edgerton and Carpenter (1923, Allyn & Bacon). In the preface the authors state that "Much of value is acquired from oral expression. The numerous oral exercises in the book help the pupil to gain these values and also the power which comes from performing mathematical processes quickly without the use of paper and pencil." There are oral lists throughout this text.

From the fact that so many texts have few oral exercises and that so few authors mention oral work as one of the strong features of their books it appears that the uses of oral work have been largely over-looked or under-rated. Let us enumerate and discuss some of these uses of oral algebra.

First, the development of much of the theory may be accomplished by oral work with very simple data. Oral work should in general precede written work. Problems involving one or two simple principles are more applicable in beginning each new topic than complicated problems, and such problems can be treated orally. In this way the complications of computation and manipulation do not distract attention from the principles. Then when principles and methods have been made clear and these are well understood, the data may be made more complex and the work written.

Second, oral algebra is of especial advantage in cultivating the idea of numbers, of literal representation, and an intelligent grasp of the numerical relations and processes. In written work there is a constant tendency among students to juggle figures regardless of aim or reason, to use any method or any operation which will lead to a plausible answer. In simple oral work it is less possible for the pupil to wander aimlessly among figures and operations. When the pupil is without his pencil, he just cannot juggle senselessly. In oral work it is easy for the teacher to express processes in terms of the operations performed. By a frequent use of "Why", the teacher may get an expression of principle from the pupil. Sometimes it is expedient to demand a principle or a reason for every step in a solution of an

algebra problem just as in the proof of a geometric theorem. In the early weeks of algebra, when dealing with literal representation and the fundamental operations, a very large part of the work is very effective if done orally. In fact very few pupils need a pencil in most of the work in addition, subtraction, multiplication, and division of monomials. Moreover, without the pencil, these pupils are more apt to think of what they are doing.

Third, oral algebra is of aid in developing clearness of speech as well as clearness of thought. The pupil is very soon able to discuss in clear and direct words the short oral problem and gains a precision in expression which is a good basis for handling the discussion of the more complicated written problems. The language value of algebra is large. Because one can state exactly a number of given conditions and the corresponding results by a very concise algebraic equation, we find every branch of science using the language of mathematics in a constantly increasing degree. The language value of mathematics is cultivated to a great extent by oral work. Expression clarifies meanings; it is only as the pupil can express himself definitely and accurately that he really has a mastery of the work.

Fourth, the oral exercise affords an excellent opportunity to develop the power to grasp a problem clearly and to make simple inferences. The pupils may easily be directed to make numerous observations and generalizations from very simple oral work. The class discussion is one of the important features of a mathematics class, for it is here that the teacher has a chance to direct the class into reflective thinking and generalization.

Fifth, oral work can be used for rapid drill work. Drill, to be most effective should be snappy and animated. With simple oral exercises many principles may be recalled and drilled on, in a short period of time without the deadening monotony of a long period of drill. Principles are fixed by frequent repitition, so frequent drill is essential. If the drill is for manipulation only, written work suffices but if it is to be a drill including practise in expression or in the correct use of algebraic language, the oral exercise has a decided advantage over the written.

Sixth, closely connected with this drill work is the use of oral algebra to review or recall earlier work just before introducing some similar or related topic in the advance work. For example

a very few minutes spent in oral review of addition of signed numbers of monomial form gives the class a basis for the new topic; addition of polynomials. A few minutes oral review is a good way to make the close contact with past experiences, which is necessary in learning or developing. It is possible to recall some previous topic with the maximum number of students actually taking part in the work and in the minimum amount of time.

Seventh, if we agree that knowledge is well organized in the degree to which it may readily be summarized, then the review at the end of any division of subject matter is one of the most important parts of the work. To conduct a review in an interesting and effective way, to lead the pupil to survey the material covered in such a way that he may see it related to a few main facts, is indeed, one of the biggest problems of teaching any school subject. If a well graded list of oral problems is at hand it may be used for the class period of review and serve as a worth-while method of variation.

Eighth, the most interesting and important use of oral algebra, I set forth as the translation problem. The best oral lists in all the books I have examined are the lists on translation of English phrases and sentences into algebraic expressions and equations. This is the type of exercise which first interested me in oral algebra. To give the pupil the ability to follow direction exactly, to translate such expressions as "find the sum of", or "a number increased by three", into the fundamental operation of addition, to give him the power to attack with confidence any word problem, derive algebraic representations for the unknowns and obtain the correct equation,—to give him such power is one big objective of first year algebra. Algebra is a symbolic language for expressing thought and with beginners it must be translated. The majority of our pupils are soon able to perform an operation with some degree of accuracy if they are told when to do it. But they lack the ability to see the problem, to analyze the situation and reduce it to a series of simple fundamental principles. To enable a pupil to translate an English problem to its algebraic equivalent, to enable him to make a plan for a problem and then follow it, is of paramount importance. Examples of pupils failure to "see the problem", or "get the question", are all too numerous. For example, ordinary prob-

lems in subtraction are commonly mis-interpreted. If the subtrahend is placed below the minuend, the pupil can perform the operation correctly, after he has learned to subtract. If the problem is written "Subtract the second polynomial from the first", and the subtrahend and minuend are given in one line, there is usually no difficulty, for the pupil is accustomed to seeing the subtrahend after he sees the minuend. Change the problem to the form, "Subtract (X-Y) from (3X+2Y)", and many students will consider the second term mentioned as the subtrahend regardless of the directions given. If specific practise is given in translation, if pupils are taught that the subtrahend follows the words "subtract", "minus", or "decreased by", this mistake in setting up the problem is largely eliminated. It is this common fault of incorrect reading, of mis-interpretation of the problem, that can be overcome by much oral work, by the special exercise in translation. Most of the texts I have examined begin with a chapter on algebraic or literal representation, instead of beginning immediately with an introduction of signed numbers, as many texts formerly did. An examination of the translation lists in various texts shows that for the most part the material is very good, though not always sufficient in amount. If much time and care is given to specific practise on translation at the outset, and if thought or word problems are always preceded by extensive work in translation, the class is much better able to follow directions, and is prepared to handle equations, for equation forming is simply the translation of the problem as written in English, into algebraic terms.

Uses in Advanced Algebra

In advanced Algebra or the third semester work, the uses of oral algebra are much the same, with stress placed on the review feature of this method. An abundance of oral exercises covering the main topics of first year algebra facilitates the review of that work, so that teacher and pupils can pass quickly over the parts which are well remembered and locate and emphasize the principles that have been forgotten.

Before discussing the disadvantages of oral work, let me tell somewhat in detail just how and when I use oral work, in the hope of presenting some really practical suggestions on methods. We have enough school copies of the supplementary text, Gol-

den and Fenno, so that each student may have one when they are to be used in class. When these texts are to be used they are distributed to the class just as are the class-room editions of a Cæsar text. By this method the material is always fresh and is oral for all the pupils because they have had no chance to work it out ahead of the class period. The amount of time and its position in the class hour depend on the purpose in using the oral text on the day in question. It may be that five minutes of rapid drill on material already fairly familiar will serve to round off a topic before developing a new concept. Often after the development and assignment of material for the following day, ten minutes of oral exercises will fix the new ideas for the pupils and will enable them to grasp the work for the following day in such a way that each will prepare his advance work intelligently and independently. In case of review over several topics preceding a test, we have used as much as twenty-five minutes for oral work in class. If I find that the whole class is weak at some point either in manipulation or in the discussion and generalization, we may turn to our oral algebra for re-enforcement. Considering the number of uses I have suggested for oral work, it might seem that our work is largely oral. This is not the case, however, for we have not used the text daily nor with any degree of regularity. On an average we use the oral work about twice a week for periods of ten or fifteen minutes. In addition to this we have daily oral discussions of our written work. Because of the many ways in which its use may be varied, there is a possibility of using it to advantage almost daily but if used daily in the same way, at the same time, and for the same purpose, it would doubtless lose much of its value.

Is there an advantage in having a sufficient amount of work in the text that is suitable for oral work, or is there a greater advantage in using the supplementary text? For convenience and to save the teacher time and trouble in organization it is perhaps desirable to use a text which includes many problems of all types and for all purposes. For the in-experienced teacher, the organization of the course offers a real problem. The first time he teaches a course, the new teacher finds it difficult to analyze the class and the work in such a way as to be able to introduce additional material, either oral or written, at the correct points.

There is often the additional expense and inconvenience in using a supplementary text.

I have preferred the supplementary texts for a number of reasons. It offers me greater freedom of organization. I may expand the work by introducing a considerable amount of outside material on certain topics and follow the text rather closely in others; whereas with a more complete text I am apt to follow along blindly without so much definite organization and revision. Then there is an advantage in having a pupil familiar with two books instead of one; of working from various presentations. We have been especially interested in developing a worthwhile Mathematics shelf in our High School library and in fostering a use of these books with a view to overcoming the popular idea that all there is to algebra is to solve ten problems daily, or in Geometry to prove a theorem. So I consider the use of a supplementary text one step in that direction. When separate texts are passed out for use in class the material is not so familiar, the work is more truly oral, and consequently more enthusiasm is shown.

Disadvantages of Oral Algebra

There are doubtless numerous disadvantages in the use of oral algebra. If used too frequently and for drill only it becomes very monotonous. There are many of us who object to so much drill, feeling that concepts, generalizations, in fact, thought values themselves, are sacrificed to manipulation by too strong an emphasis on drill. So though the drill element may largely predominate in oral algebra, I answer this objection by emphasizing three points:

First, if the drill in manipulation is well organized and quickly handled, more time is saved for the emphasizing and applying fundamental concepts of mathematics.

Second, that oral algebra, though often short and snappy in form, is not all manipulation drill, or at least, need not be. Drill, though often considered so, does not mean just symbol manipulation. Pupils can be drilled in reflective thinking, also. Drill in reflective thinking, in discussion, in the formulation of original problems grows out of oral work entirely.

Third, oral work creates confidence. If the pupil learns that he is able to do much in a short period of time, that he is able

to express his results clearly, he grows confident and is encouraged to attack any problems which confront him, even when these problems increase in difficulty.

Another difficulty in using oral algebra and in the oral discussion of written work is that material which may do admirably for oral work for some students, can not be so handled by others, for we do not escape the problem of individual differences, even here. I meet this difficulty by not making any hard and fast distinction between oral and written work. The two are complementary phases of the same subject and the same instruction, hence it is frequently desirable to pass from one to the other. One of the most favorable features of oral work is that the teacher can often give help or suggestions just at the point and in the degree in which it is needed. To the pupil who has trouble in formulating oral responses, much help can be given by the teacher setting down the results on the board as the pupil talks. The pupil is helped to continue by this visual aid, but he is not so dependent on writing as he would be with chalk or pencil in his own hand. My suggestion then, in taking the inarticulate pupil over the gap between written and oral work is to go from written work by the student, to the exercises where the student talks and the teacher writes, and then gradually arrive at the point where there is no writing of results by either pupil or teacher. In our classes we continually go from the method of oral discussion to a method that we call "chalk talking". The "chalk talk" method means that together we are making a plan, for the most part orally, and that one of us is acting as recorder or secretary by setting forth the important steps on the board. At first, until the class has learned to use this method, the teacher works at the board developing the work, according to student direction. Then as soon as possible the teacher has one of the class stay at the board and do the written work which the others dictate. There are many pupils who cannot get the point quickly enough to act as the recorder and it is better to choose those who are efficient for this work so that class time will not be wasted. Those who are slower can be chosen to take part in the work where there is little to be written and their slowness will not hinder the class so much. Finally the pupil may reach the point where he himself can do the talking and writing at the same time, and

carry on a discussion of a problem just as he works it out. It is more difficult for a student to use this method and a majority of the class may never reach this last stage. However, it is not so difficult to develop the method to the point where most of the students are able to conduct the board work according to direction from the pupils and teacher. Oral work seems to lead naturally to these valuable "chalk and talk" exercises. Whenever the work is conducted by oral methods, it is wise to turn to written methods, where it will advance the work, either in clearness or speed.

Conclusions

Oral algebra includes all work not written, whether it be the short oral problem or oral discussion of written work. A survey of the field shows that few authors have considered this phase of teaching, at least to any great extent.

The uses of oral algebra which are of especial value are listed again:

1. Much of the theory is better developed by oral algebra, so that complications of manipulation do not distract the attention from principles.

2. Oral algebra aids in cultivating the idea of numbers and of fundamental operations, without the temptation to juggle figures, which often accompanies written work.

3. Oral algebra gives the pupil precision of speech, and expression clarifies meanings.

4. In oral work there is opportunity for reflective thinking, generalization, a chance to grasp the problem and then make simple inferences.

5. Rapid drill in manipulation is facilitated by oral work.

6. Oral algebra is a means of reviewing past work, before introducing new work; it is a method of establishing a point of contact with past experiences.

7. Review at the end of a topic may be largely oral.

8. Oral algebra is important in developing the ability to translate, to see the problem, to understand and follow directions, to set up equations; in short, the language value of algebra is cultivated almost entirely by oral work.

9. In advanced algebra, review and drill of the fundamentals may be accomplished in a short period of time by much oral work.

We use the supplementary oral texts about twice a week for ten or fifteen minute periods, but the extent of daily discussion of written work extends the amount of oral work in our courses, so that it exceeds the written as far as class periods are concerned.

To keep oral work from being monotonous, it should not be mere manipulation but should include the variety of uses suggested. To be effective, oral work is best if combined with written work. The method of combination which I have found effective is that described as the "chalk talk" method. In the use of oral algebra, I have been trying to emphasize the real values of algebra, to make it vital, interesting, and practical for the students.

AN EXPERIMENT IN TESTING AND CLASSIFYING PUPILS IN BEGINNING ALGEBRA[1]

By C. M. AUSTIN
High School, Oak Park, Ill.

This experiment was started about five years ago. At that time the grade schools began sending their estimate of the pupils' ability to the high school office. Each pupil was rated as being A, B or C in ability. This classification was accepted at its face value and pupils placed in three groups. This grouping was not at all satisfactory from our standpoint as the grade teachers seemed to rate the pupils according to behavior or personal preference or achievement in some other study.

Next the Stone Reasoning Test in Arithmetic was used to supplement this judgment of the grade teachers. This plan also was unsatisfactory.

Then in the Spring of 1921, the Otis Intelligence Test[2] was first used. Many of you will no doubt remember that at the meeting of the Central Association of Science and Mathematics Teachers in November, 1920, Professor Buckingham, then connected with the University of Illinois, read a paper designed to prove that no relation existed between achievement in mathematics and general intelligence. The writer did not believe his conclusions and resolved to disprove them, if possible. So in February, 1921, at the writers' own expense, the Otis Test was given to two hundred entering freshmen. Two purposes were in view—first, to see if some relation between general intelligence as revealed by the Otis Test and School Grades in Algebra could be discovered; and second, to see if this test would give a sound basis for classifying pupils for the study of algebra and other school subjects. The results of the test are shown on Chart I.

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[1] Read before the Mathematics Section of High School Conference at Urbana, Ill., November 23, 1923.

[2] The Otis Test is a composite of ten tests: Following Directions, Opposites, Disarranged Sentences, Proverbs, Arithmetic, Geometric Figures, Analogies, Similarities, Narrative Completion, Memory.

A pupil's score is the sum of the points made in the ten tests. By studying the results obtained in testing thousands of pupils, norms have been etablished for the different ages.

One hundred has been arbitrarily fixed as normal with 50 per cent of the individuals below and 50 per cent above.

A pupil's Index of Brightness is 100 plus the difference between his total score and the established norm for his age. If an individual's total score is 142 and the norm for his age is 124, then his Index of Brightness is 118, indicating that he is 18 points above normal.

CHART I

INDEX OF BRIGHTNESS

	0-19	20-39	40-59	60-79	80-99	100-119	120-139	140-159	160-179	180-199	Total
90-100	--	--	--	--	1	3	8	11	7	3	33
80-89	--	--	--	--	4	9	20	16	6	2	57
70-79	--	--	1	6	8	15	26	9	--	--	65
60-69	--	--	1	2	2	7	2	--	--	--	14
50-59	--	--	1	3	4	6	1	--	--	--	15
40-49	--	--	--	--	1	3	--	--	--	--	4
30-39	--	--	--	2	--	1	--	--	--	--	3
20-29	--	--	--	--	--	1	1	--	--	--	2
10-19	--	--	--	1	--	--	--	--	--	--	1
0-9	--	--	--	--	1	--	--	--	--	--	1
	--	--	3	14	21	45	58	36	13	5	195

ALGEBRA, FEBRUARY, 1921

$r = 0.54 \pm .03$

38 below normal—20 passed, 18 failed.

40 failed—22 above normal, 18 below.

For English, $r = 0.64$

For Science $r = 0.59$

Several significant things need to be pointed out:

1. *Forty pupils failed to secure a passing grade at the end of the semester. This is practically 20%, a figure which is too high. Of those failing, 22 were above normal and 18 below.*

2. *The number of failures corresponds closely to the number of those who are indicated by the test as being below normal in intelligence. Thirty-eight were below normal of whom 20 passed and 18 failed.*

3. *Manifestly, some of these individuals are displaced. Their achievement in algebra does not correspond to the Index of Brightness given by the Otis Test. Their grades need explanation. Some of these pupils were in the writer's class. In other instances the teachers gave their opinions for the failure or success of the pupils. A few of these explanations follow.*

Pupil A___ Algebra 5 I.B. 98 —Almost normal by the Otis test but did absolutely nothing in Algebra. No effort whatever.

Pupil B___ Algebra 45 I.B. 108 —Slightly above normal, but made no effort to master Algebra. Lazy.

Pupil C___ Algebra 75 I.B. 83 —Below normal, but very attentive and faithful in her work. Always anxious to please the teacher. Hard worker.

Pupil D___ Algebra 60 I.B. 112 —Above normal, but very lazy. Wore embroidered jacket. Too much money. Did not see need of work.

Pupil E___ Algebra 70 I.B. 87 —Below normal, but a steady, faithful worker, never slighted any work assigned by teacher.

Pupil F___	Algebra 80	I.B.	92	—A little below normal, but a hard worker. Willing to stick to a problem until it was mastered. Also active in class discussions.
Pupil G___	Algebra 65	I.B.	117	—Did good work for a time, then slowed down; not much ambition.
Pupil H___	Algebra 80	I.B.	92	—Below normal but a faithful worker. Always did all required work. Frequently came to teacher after school for explanation.
Pupil I____	Algebra 80	I.B.	95	—Faithful, hard, conscientious worked,
Pupil J____	Algebra 90	I.B.	94	—Very faithful, attentive, conscientious.
Pupil K___	Algebra 70	I.B.	96	—Very much like pupil J.
Pupil L____	Algebra 70	I.B.	97	—A hard worker.
Pupil M___	Algebra 70	I.B.	64	—Very faithful and willing to work.
Pupil N___	Algebra 70	I.B.	52	—A repeater, but willing to work.
Pupil O____	Algebra 60	I.B.	112	—Lazy.
Pupil P___	Algebra 40	I.B.	101	—Just normal but very lazy is now doing fairly well. Plays football and so has to keep up in studies.
Pupil Q___	Algebra 20	I.B.	120	—A nervous, flighty chap, easily distracted. No concentration at all.
Pupil R___	Algebra 20	I.B.	83	—A conscientious, hard worker.
Pupil S____	Algebra 65	I.B.	122	—Immature, playful.
Pupil T___	Algebra 93	I.B.	109	—A slow but very accurate worker. Her slowness was a handicap in Otis Test, but all questions attempted were correctly answered.
Pupil U____	Algebra 30	I.B.	107	—Lazy, Made no effort at all.
Pupil V___	Algebra 60	I.B.	116	—Started well but was absent a great deal.
Pupil W___	Algebra 60	I.B.	111	—Very peculiar girl. Timid and diffident. Often refused recitation on that account.
Pupil X___	Algebra 70	I.B.	86	—Very faithful and consistent worker, Always willing and cheerful. Very neat.
Pupil Y___	Algebra 70	I.B.	77	—Very similar to pupil X. Is still in school, has never failed in any subject.

4. *The coefficient of correlation is* $0.54 \pm .03$. *This means that the true value of r lies between 0.51 and 0.57.*

In October, 1921, the Otis Test was given to all entering pupils. The results appear on Chart II. The correlation coefficient is lower, but still high enough to show that general intelligence and ability to achieve in algebra are related.

CHART II

INDEX OF BRIGHTNESS

	0-19	20-39	40-59	60-79	80-99	100-119	120-139	140-159	160-179	180-199	Total
90-100	--	--	--	--	--	3	8	10	4	1	26
80-89	--	--	--	3	14	24	53	20	9	3	126
70-79	--	--	--	10	21	63	49	17	1	--	161
60-69	--	--	--	3	8	11	6	1	--	--	29
50-59	--	1	1	7	5	17	4	1	1	--	37
40-49	--	--	--	2	4	4	--	--	--	--	10
30-39	--	--	--	1	2	1	--	--	--	--	4
20-29	--	--	--	--	1	1	--	--	--	--	2
10-19	--	--	--	--	--	--	--	--	--	--	
0- 9	--	--	--	--	--	--	--	--	--	--	
-	--	1	1	26	55	124	120	49	15	4	395

ALGEBRA, SEPTEMBER, 1921

$r = .54 \pm .03$

38 below normal—20 passed, 18 failed.

40 failed—22 above normal, 18 below.

Again about 20% failed to make a passing grade for the semester. Thirteen of those failing are found in the lower right hand corner of the diagram, with an Index of Brightness over 120. The reasons assigned by the teachers for eight of these failures follow:

Pupil A___ Algebra 65, I.B. 123 —A rank slacker.

Pupil B___ Algebra 55, I.B. 126 —Lazy to the last degree—Didn't care.

Pupil C___ Algebra 60, I.B. 131 —Had an after school job.

Pupil D___ Algebra 50, I.B. 128 —Immature, timid, made no success in any subject.

Pupil E___ Algebra 50, I.B. 138 —Extremely lazy and indifferent. Slovenly in personal appearance and work.

Pupil F___ Algebra 50, I.B. 144 —Also lazy. Afterward did fairly well.

Pupil G___ Algebra 65, I.B. 165 —Absent one or two days each week.

Pupil H___ Algebra 60, I.B. 127 —Just plain indifference. Failed in other subjects.

Of those pupils who made passing grades, 48 were below normal according to the Otis Test. A few of the teachers' reactions follow:

Pupil A___ Algebra 85, I.B. 97 —Very conscientious. Did daily work faithfully.

Pupil B___ Algebra, 70 I.B. 75 —A plodder. Did a great deal of work outside of class.

Pupil C___ Algebra 85, I.B. 88 —Very conscientious and faithful worker.

Pupil D___ Algebra 85, I.B. 68 —Faithful in daily work.

Pupil E___ Algebra 70, I.B. 93 —Had taken part of Algebra in another School.

Pupil F___ Algebra 70, I.B. 79 —Had a great deal of outside help.

Pupil G___ Algebra 85, I.B. 99 —Paid good attention in class. Conscientious, hard worker.

Pupil H___ Algebra 85, I.B. 99 —Same as above.

Pupil I____ Algebra 70, I.B. 96 —Hard worker, often sought help from the teacher.

Pupil J____ Algebra 80, I.B. 91 —Very hard worker. Extremely interested in Algebra.

Pupil K____ Algebra 80, I.B. 77 —Very perservering. Sought help from the teacher nearly every night.

Again in February, 1922, the Otis Test was given to the entering class. The results are shown in Chart III.

CHART III

INDEX OF BRIGHTNESS

	0-19	20-39	40-59	60-79	80-99	100-119	120-139	140-159	160-179	180-199	Total
90-100	--	--	--	--	--	--	8	11	2	--	21
80-89	--	--	--	--	4	14	14	18	1	--	51
70-79	--	--	1	3	8	27	21	8	2	--	70
60-69	--	--	1	--	2	4	2	1	--	--	10
50-59	--	--	--	1	10	3	1	1	--	--	16
40-49	--	1	--	--	2	2	1	--	--	--	6
30-39	--	--	--	1	1	--	--	--	--	--	2
20-29	--	--	--	--	--	--	--	--	--	--	
10-19	--	--	--	--	--	--	--	--	--	--	
0- 9	--	--	--	--	--	--	--	--	--	--	
	--	1	2	5	27	50	47	39	5	--	176

ALGEBRA, FEBRUARY, 1922

34 failed—15 above normal, 19 below
35 below normal—15 passed, 20 failed
$r = 0.54 \pm .04$
For English, $r = 0.45$

Thirty-four failed for the semester, of whom fifteen were normal or above and nineteen were below. Thirty-five are below normal in intelligence of whom 15 passed and twenty failed.

In September, 1922, 392 entering pupils took the test of whom 335 stayed in school through the semester. The results are shown on Chart IV.

CHART IV

INDEX OF BRIGHTNESS

	0-19	20-39	40-59	60-79	80-99	100-119	120-139	140-159	160-179	180-199	Total
90-100	--	--	--	--	2	3	9	13	2	--	29
80-89	--	--	--	1	7	22	29	31	6	--	96
70-79	--	--	3	7	34	49	34	12	5	1	145
60-69	--	--	--	5	7	6	4	2	--	--	24
50-59	--	--	1	2	6	6	4	1	--	--	20
40-49	--	--	1	--	2	6	2	1	--	--	12
30-39	--	--	1	1	2	--	1	--	--	--	5
20-29	--	--	--	--	1	--	1	--	--	--	2
10-19	--	--	--	2	--	--	--	--	--	--	2
9-9	--	--	--	--	--	--	--	--	--	--	
	--	--	6	18	61	92	84	60	13		335

ALGEBRA, SEPTEMBER, 1922

85 below normal—54 passed, 31 failed
65 failed—31 below, 34 above
$r = 0.40 \pm .03$

Sixty-five failed to secure a passing grade, of whom 31 were below normal and thirty-four above. Eighty-five pupils were below normal, of whom 54 passed and thirty-one failed.

The coefficient of correlation for this group was $0.43 \pm .03$.

In February, 1923, another group of 206 took the test. Forty-four failed, of whom 23 were below normal and 21 above. Only 35 of the whole group were below normal, of whom 12 passed and 23 failed. The coefficient of correlation for this group is $0.53 \pm .03$. These results are shown on Chart V.

CHART V

INDEX OF BRIGHTNESS

	0-19	20-39	40-59	60-79	80-99	100-119	120-139	140-159	160-179	180-199	Total
90-100	--	--	--	--	--	3	9	11	5	--	28
80-89	--	--	--	1	6	13	21	18	5	3	67
70-79	--	--	--	1	4	21	26	14	1	--	67
60-69	--	--	--	2	5	6	4	1	--	--	18
50-59	--	--	--	6	6	2	5	--	--	--	19
40-49	--	--	--	1	1	1	1	--	--	--	4
30-39	--	--	1	--	--	--	1	--	--	--	2
20-29	--	--	--	1	--	--	--	--	--	--	1
10-19	--	--	--	--	--	--	--	--	--	--	
0-9	--	--	1	12	22	46	67	44	11	3	206

ALGEBRA, FEBRUARY, 1923

$r = 0.53 \pm .03$

35 below normal—12 passed, 23 failed

44 failed—23 below, 21 above

For English, $r = 0.50 \pm .03$

The coefficient of correlation between the grades in English and the Otis Test is $0.50 \pm .03$.

The class that entered school in September, 1923, also took the test. Their November grades in algebra were correlated with the Otis Test scores. Chart VI shows the result.

CHART VI

INDEX OF BRIGHTNESS

	0-19	20-39	40-59	60-79	80-99	100-119	120-139	140-159	160-179	180-199	Total
90-100	--	--	--	2	1	9	12	14	8	2	48
80-89	--	--	--	4	20—	35	39	27	5	2	132
70-79	--	--	3	8	41	41	42	9	1	1	146
60-69	--	--	2	6	8	12	4	1	--	--	33
50-59	--	--	2	1	7	2	--	1	--	--	13
40-49	--	--	--	--	3	--	--	--	--	--	3
30-39	--	--	--	--	--	--	--	--	--	--	
20-29	--	--	--	2	--	--	--	--	--	--	2
10-19	--	--	--	--	--	--	--	--	--	--	
	--	--	7	23	80	99	97	52	14	5	377

ALGEBRA, NOVEMBER, 1923

110 below normal—59 passed, 51 failed—

51 failed—31 below, 20 above

$r = 0.46 \pm .03$

Fifty-one or 13.5% failed to make a passing grade, of whom 18 were above normal and 33 below. One hundred ten pupils are shown to be below normal, of whom 59 obtained a passing grade and 51 failed. $r = 0.46 \pm .03$.

At the present time, Oak Park pupils are classified into three groups, A, B and C according to the judgment of the grade teachers. Soon after entering the Otis Test is given. Then as soon as the teachers are able to judge accurately, adjustments are made, based upon the Otis Test scores and the pupils' achievement in algebra. Each time many are found to have been wrongly classified by the grade teachers. In September, 1922, one whole section classed as B by the grade teachers was only C in achievement. Sixteen of them had I.B.'s of less than 113. Another section rated as B had 12 individuals whose I.B.'s were below 115. This section was really of C ability only. Forty-nine others were found to be misplaced, of whom twenty-five were transferred from C to A and B sections and twenty-four from A and B to C sections. Of the twenty-five all but one made good in the higher group. Some have to be transferred from A and B sections to C sections and vice versa. In nearly every instance those going up have made good. The ones going down are in classes correspondng more nearly to their capacities and they are, therefore, happier and doing better work.

Teachers Technique for C Sections

1. *Every new principle should be carefully developed and explained. This process must be repeated until all have grasped the idea.*

2. *Frequent short tests of one or two problems should be given to discover whether or not the pupils have mastered the principle.*

3. *Teach and Test—Teach and Test again.*

4. *All problems in which the principle is hidden by unrelated material should be omitted.*

5. *Most of the complicated problems should be omitted. Some problems should be difficult enough to require application of the principle involved, else the principle will not be mastered.*

6. *Most of the work should be done in class room under supervision of teacher. A minimum of recitation and a maximum of teaching should be the motto.*

7. *Assignments of home work must contain no new principles.*

8. *Assignments of home work must be well within the ability of pupils.*

9. *Teacher must use every precaution to keep the pupils confident.*

10. *Never send a C class out of room without knowing just what to do.*

11. *Discourage help from other pupils and from parents and friends. They tell—telling is not teaching.*

12. *Make class period largely a work period. Explain new principles, drill on problems applying them.*

13. *Urge pupils to express themselves in algebraic language. Chalk and talk—giving all principles involved. A principle or process is not mastered by a pupil until he can use it in expression.*

14. *Have all work checked, so that the pupil can know that his work is correct. It also helps to keep the pupil confident.*

15. *Urge pupils to be critical of each others' work in friendly way.*

16. *Let original thinking with numbers and symbols be the aim. Mastery of a few principles is better than a superficial study of many.*

Advantages of Classification

1. *It enables the brilliant pupils to forge ahead. There are no dull moments in recitation period.*

2. *It enables the school to develop leaders. It promotes independent thinking.*

3. *The same amount of explanation answers for all the pupils in the section.*

4. *The weaker pupils being together have courage to ask questions and take part in discussions. They never do this if in section with strong students.*

5. *Thus, leaders among the weaker pupils are devoleped also.*

6. *Pupils in C sections have greater frankness in asking questions.*

7. *It allows of more development of topics by class and teacher without injury to bright pupil.*

8. *One of the greatest faults of mixed classes is the injury to the brilliant pupil through marking time while the slow ones "catch on"—classification prevents that.*

9. *Classification should prevent failures. If the subject matter is rightly selected and if the instruction is suited to the class there should be very few failures in C sections.*

Conclusions and Observations

1. The grade school teachers are unable to correctly classify pupils for work in high school subjects. Out of an entering class of about 400 pupils in September, 1923, 57 individuals were misplaced; 34 had to be transferred from A and B sections to C sections and 24 from C to A or B groups. The Otis Test would have placed them all correctly.

2. While many pupils with high Otis, I.B.'s made low grades in algebra, no pupil with a low I.B. succeeded in making a high grade in algebra. They may make a fairly high mark for a month or so, but soon drop to a lower level.

3. The Otis Test is not an infallible guide in classifying pupils for achievement in algebra. It can select those who are capable mentally of succeeding, but it cannot measure the personal characteristics so necessary to success. Often an individual with a high I.B. will be so lazy and indifferent that only failure can result. Last year the boy with the highest I.B.-198 was absent from school about two days each week. He barely made a passing grade. Other individuals with lower I.B.'s but with the capacity and willingness to work continually made high marks.

4. The test is a valuable guide to teacher and supervisors in securing better work from the pupils. The writer has in mind a boy who at first was so timid that he seemed to be unable to do successful work. His I.B., however, indicated a fair degree of ability. By careful questioning and encouraging attention, he developed rapidly and made a fair grade. He is now a junior and is doing well. Without the test score, he might have been neglected entirely.

5. Any pupil having an I.B. of 100 or more may be sure of a passing grade in algebra if he is willing to assume the proper attitude and put forth the proper effort, since practically all the teachers testify that attitude and effort are usually the controlling factors in a pupil's success. This is also shown by the fact that many individuals with I.B.s as low as 80 have succeeded fairly well.

6. Of those who withdraw from school saying that the course is too difficult and that they do not like school, etc., practically 5 out of 7 are below normal as indicated by the Otis Test score.

7. Of the 57 individuals, juniors, sophomores, freshmen on the honor roll for November, 1923, only two had I.B.s below 120. All ranged high. Twenty-eight freshmen were on the honor roll during the year 1922-23. All had high I.B.s but one who was six points below normal.

8. While the correlation coefficient is not as high as the writer would desire, yet it is sufficient to show that there is a considerable relation between general intelligence and ability to achieve success in algebra. Dr. Rugg in his book on *Statistical Methods Applied to Education* says that correlation can be regarded as "being markedly present or marked, when r ranges from .35 or .40 to .50 or .60." So, from this series of tests it can be said that correlation is markedly present. This means that pupils with a high Otis I.B. will most likely be able to make a high grade in algebra. Since the number of cases is large and the probable error is very small the coefficient is fairly reliable.

9. The equations of the lines of regression on Chart V are $x = .53y$ and $y = .53x$. The first one means that a unit deviation in the Otis I.B. is accompanied by a deviation of .53 as much in the algebraic grade. The second means that a unit deviation in algebraic achievement is accompanied by a deviation of .53 as much in the Otis I.B.

10. By making use of Pearson's equation for the line best fitting the data, it can be determined what grade in algebra ought to accompany a given Otis I.B. This equation is $y_1 - \bar{y} = r\frac{\sigma y}{\sigma x}(x_1 - \bar{x})$, $\bar{y}$ and $\bar{x}$ are the mean values of the columns and rows; σy and σx are the standard deviations of the two distributions, r is the correlation coefficient, y_1 and x_1 are

the absolute values of particular individuals. Using real values from Chart V and substituting these values in the formula above the following table is constructed:

Otis I.B.	Algebra Grade
50	37.24
60	42.54
70	47.84
80	53.14
90	58.44
100	63.74
110	69.04
120	74.34
130	79.64
140	84.94
150	90.24
160	95.54
170	100.84

11. The C section is the great problem. The old material will not suffice for those pupils. The old methods of presentation will not succeed. It is the writer's opinion that if the C pupil can be taught to think and express himself with the elements of the subject, it is enough. Far better than not to have him think at all. Far better than to drag him over the old course. Hatred of the subject and failure can only result.

12. The teachers in the department of mathematics are strongly in favor of classification. They believe with the writer that it brings the greatest good to the greatest number of pupils.

DISCUSSION

An Interesting Circulating Decimal. As matter of instruction for our children, the subject of circulating decimals is now ruled out. Our teachers, however, are still expected to have a working acquaintance with its simpler forms. Hence this note.

By the usual method we may evaluate .3 and convert it into the common fraction $\frac{1}{3}$. The test of our findings lies obviously in the fact that we are able to reconvert $\frac{1}{3}$ into .3. The same procedure will apply to .4, .5 and so on through 8. In each case the acid test must be met,—the common fraction must be reconverted into the given circulating decimal.

What shall we make of the circulate .9?

Evaluating it by the usual method we get the answer 1, which obviously cannot be reconverted into .9.

Is there any fraction, or type of fraction which will yield the circulate .9? If so, what is it?

It is not difficult to "see" that the value of .9 is one. By comparing the circulates through .8 we can by legitimate mathematical generalization "see" that each of these approaches a certain limit, namely $\frac{1}{9}, \frac{2}{9}$, and so on through $\frac{8}{9}$; similarly the circulate .9 approaches as *its* limit $\frac{9}{9}$, and *approaches its limit* 1 *more closely than any other quantity that can be designated or expressed.*

This description of .9 may almost be deemed its definition.

Take now any fraction of which the numerator is equal to the denominator, say $\frac{2}{2}$. Its value is 1.

Let us now regard the numerator 2 as a limit. The quantity that can approach it more closely than any other is 1.9.

Substituting for the numerator this alter ego we have $1.\frac{9}{2}$, which yields the circulate .9.

The circulate .9 can always be derived from any expression of the form m-m when for m in the numerator we substitute m—1 +.9=(m—(1—.9)). For the moment we may confine m to mean a positive integer.

When the fraction is of the form m-n, and m is a multiple of n, we derive the circulate .9 in combination with an integer. For instance $\frac{8}{2}$; which when the substitution is made becomes $7.\frac{9}{2}$=3.9.

(Similarly the circulate .9 may be derived as part of a decimal fraction by making an expression m-n when mis prime to n and treating it as above. For instance $\frac{3}{4}$, which then becomes $2.\frac{9}{4}$ and yields .749).

In fine the circulate .9 may be said to be the series which approaches its limit 1 closer than any other assignable value.

It is derivable when its limit, added to the rest of the numerator of a fraction, is equal to, or a multiple of, the denominator of the fraction. It is derivable, therefore from an indefinite number of fractions. That it must itself be used as part of the expressions which produce it, is one of its characteristics.

SIMON HIRSDANSKY

New York City.

Classification of pupils in algebra.: This semester, in two nine-one classes, one nine-two class, and three eleven-one classes of Algebra, the attempt is being made to accomplish the minimum, the average, and the maximum assignments. In every class, the students have been divided into three groups by the aid of the Thorndike-McCall Reading Test, the Terman Group Test, speed tests which were devised by the teacher, and also by the subjective marking of the instructor.

The students have been assigned code numbers, and percentile curves have been made for the third set of tests which is mentioned above, in order to show the student his relative mark as compared with those of the other students who are taking his subject. The assignments are made according to the abilities of the studetns, and in all cases the mastery of them is required. In addition to the enriched curriculum for the third (highest) group, special outside work is being required.

WALTER E. HARNISH.

Fort Smith, Arkansas.

"You Never Can Tell." It is often asserted that high school pupils should not be compelled to study some certain subject when they never will have any use for it." But who is to be the omniscient prophet who can predict the career of even one child? Who has the eagle mental eye to foresee the impulse of new conditions or the influence of some powerful new motive which may come into a life.

Except the dead languages, probably no subpect has the "no-use-for-it" argument brought against it so often as mathematics.

The result of popular outcry against mathematics has led many high schools to graduate students with very little work in that line.

Indiana University still requires two years of mathematics, one of algebra and one of plane geometry, for entrance to regular standing.

Last September Dean Rothrock found a number of students who were conditioned in plane geometry. To these students he offered the opportunity of a class at the university to make up the condition. Miss Avie Burkett, a senior, mathematics major and an experienced teacher, was secured to instruct the class. Miss Burkett gives the following interesting account of the outcome of the work.

Last fall a class formed for those students who were conditioned in plane geometry. The class consisted of 15 freshmen, one special student, two sophomores, two juniors, and two seniors—a total of 22. Why was it that these people did not have their geometry requirement? Some may think that it was because they came from small high schools, others may think it was because they came from large high schools. The facts are:

Seven came from high schools of 100 or less.
Eight came from high schools of between 300 and 400;
Seven came from high schools of between 1,200 and 4,000.

This shows that the size of the high school has nothing to do with their not having had the geometry requirement. The question was asked, "How many failed in the subject in high school?" There were ten who had had some geometry, but only two of them failed; one bacuse of sickness, and one because of disinterest. The rest just dropped out after three or more days; some after one-half semester. Why do students avoid geometry? The reasons which these students gave are the same as may be given by any other group avoiding the subject.

Ten said they could not get mathematics;
Twelve said they were afraid of geometry;
Eleven said they knew they wouldn't like it;
Eight said they disliked mathematics;
Nine said they didn't expect to go to college.

There was only one who said she took all that was required. What would you expect from a class of this sort? I believe you

can imagine how a teacher must feel when she knows that every student in the class is there because he must be.

From the first day an emphasis was laid upon reasoning things out. The result was that only one student memorized the work. Others tried it but gave it up and said it was much easier to think things out and see the relation of things. It became plain to the class that the method of work is more important than the subject matter. Of course we must have both. It is interesting to know what these people really thought of geometry at the end of their course. Some thought they had profited much by it. Two freshmen and one senior said they enjoyed it more than any other course and others said it was the first mathematics they had really enjoyed.

One of the sophomores said if you look at geometry in the right way it is art. Another student said he was convinced that mathematics may be made more interesting and more fascinating than any other course. There were three students, one sophomore and two freshmen, who said it had gotten them over their self-consciousness. All said that they found their former conceptions of geometry unfounded. Two of them had had such a horror of it that they almost decided not to come to Indiana University, but thought perhaps they would not be compelled to take it. Many had the opinion that it was a hard dry uninteresting subject and one of the best students expected it to be a bore. He soon forgot that and found himself wishing he could spend hours on it. At first, some thought it would be very tiresome to go to this class every afternoon, but before long they became so interested that they forgot they were not even getting credit for it.

This class proved to be successful above the average and one of the most interesting to work with. Here may be a surprising fact to some, 13 of these students expect to take more mathematics and four would take more if time permitted, out of the 13, five expect to major in mathematics and seven to minor in it—a good record for any class.

This goes to show what foolish notions and unfounded conceptions people sometimes have of a subject.

J. Z. A. McCaughn.

Bloomington, Ind.

NEWS AND NOTES

With Mr. E. L. Thompson of Joliet Township High School presiding, the Mathematics Section of the Central Association met in the Shortridge High School, Indianapolis.

Miss Edith Inks of the Oak Park High School opened the program. For several years Miss Inks has been working on the interesting series of slides she presented. By means of these slides she shows the importance of mathematics throughout our lives. Her audience saw illustrations of the uses the greatest artists and architects and engineers throughout all the ages have made of the principles of mathematics. The Greatest Artist of all has arranged the leaves on the stems of plants in accordance with the extreme and mean ratio, and has taught the spider to weave a mathematically perfect web. It isn't surprising that Miss Inks has found her slides arousing interest and enthusiasm in her classes.

Last year Mr. W. W. Hart of the University of Wisconsin, Mr. W. D. Reeve and Mr. M. J. Newell were appointed by Mr. Gingery, last year's chairman, as a "steering committee." The committee, to quote Mr. Hart, was intended "to promote the study of, to gather data about, and to stimulate the discussion of matters essential to the organizaion of courses in mathematics in the seventh to the twelfth grades." The committee sent a questionnaire, the results of which were presented by Mr. Hart under the topic, "A Study of Problems Connected with the teaching of Mathematics."

"Hurdles—a Device for Determining the Degree of Mastery" was the particular problem which Miss Kate Wenz of Indianapolis chose in leading the discussion which followed.

Since only forty-four replies to their questionnaire were received, and as Miss Gugle pointed out, this response hardly represented the section, the committee was continued for another year. It was left to them to decide their course of action.

The Saturday morning program was made up of two very interesting discussions. Mr. W. W. Gorsline talked on the "Slide Rule in Secondary Schools," submitting a report of a committee of the Chicago Men's Mathematics Club taking up in detail the use of the slide rule in 2A geometry. Mr. Fiske Allen of

the Training School, Eastern Illinois State Teachers' College, talked on the mathematics course of the senior high school.

The report of the nominating committee appointed Friday afternoon was accepted, and the following officers were elected: Chairman, A. M. Allison, Lake View High School, Chicago; Vice Chairman, Raleigh Schorling, University High School, Ann Arbor; Secretary, Miss A. Blanche Clark, High School, Rockford, Ill.

The meeting adjourned. GERTRUDE L. ANTHONY.
Oak Park High School.

At its November meeting the Men's Mathematics Club of Chicago elected a delegate to represent the club at the next session of the National Council of Tachers of Mathematics to be held in Chicago, February 23, 1924. The club indicated its willingness to defray at least a part of the delegate's expenses when the meeting is not held in Chicago.

The members of the club believe that every organization of teachers of mathematics in the country should take similar action. If such action were taken, the meeting would take on a real national aspect and be of greater service to the teachers of our subject.

PROGRAM OF NATIONAL COUNCIL

—of—

MATHEMATICS TEACHERS

FEBRUARY 23, 1924

Meetings will be held in La Salle Hotel

10:30 A. M. Business Meeting

"A Better Use of Tests"
W. D. REEVE
University of Minnesota

3:00 P. M. Program
"Memory and Marks in Mathematics"
ETHEL LUCCOCK
Northwestern High School, Detroit

"The Laboratory Method in the Class Room"
CHARLES STONE
Chicago University High School

6:00 P. M. Banquet
Program
Dr. H. E. Slaught, University of Chicago, Presiding

"Reliability of Teachers' Marks"
RALEIGH SCHORLING
Michigan University High School, Ann Arbor, Mich.

"Teaching Pupils the Conscious Use of a Technique of Thinking"
ELSIE P. JOHNSON
Oak Park and River Forest Township High School,
Oak Park, Ill.

The program was prepared by the president, Dean John H. Minnick, School of Education, University of Pennsylvania.